AQA

AS
Mathematics

MPC1-Pure Core 1
Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, the principal examiner may suggest that we award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\sqrt{98}=7\sqrt{2}$ or $\sqrt{32}=4\sqrt{2}$	M1		
	$\left(7\sqrt{2}-4\sqrt{2}=\right)\ 3\sqrt{2}$	A1	2	
(b)	$\dfrac{**}{2+3\sqrt{2}}\times\dfrac{2-3\sqrt{2}}{2-3\sqrt{2}}$	M1		
	[Numerator =] $6\sqrt{2}-18$	A1		multiplied out
	[Denominator $=4+6\sqrt{2}-6\sqrt{2}-18$] $=-14$	B1		must be seen as denominator
	Value $=\dfrac{6\sqrt{2}-18}{-14}$ $=\dfrac{9}{7}-\dfrac{3}{7}\sqrt{2}$ **or** $-\dfrac{3}{7}\sqrt{2}+\dfrac{9}{7}$	A1cso	4	must have these two simplified fractions for **A1 cso** but may have $1\dfrac{2}{7}-\dfrac{3}{7}\sqrt{2}$ **etc** condone $\dfrac{9}{7}-\dfrac{3\sqrt{2}}{7}$ for **A1 cso** No **ISW.**
	Total		6	

NO MISREADS ALLOWED IN THIS QUESTION

(a) **NMS** $3\sqrt{2}$ scores full marks

(b) Condone multiplication by $2-3\sqrt{2}$ instead of $\times\dfrac{2-3\sqrt{2}}{2-3\sqrt{2}}$ for **M1 only if** subsequent working shows multiplication by both numerator and denominator – otherwise **M0**

An error in the denominator such as $4+5\sqrt{2}-5\sqrt{2}-18$ should be given **B0** and it would then automatically lose the final **A1cso**

May use alternative conjugate $\times\dfrac{3\sqrt{2}-2}{3\sqrt{2}-2}$ **M1 ; (**Denominator = $18-6\sqrt{2}+6\sqrt{2}-4)=14$ **B1** etc

Alternative: M1 for $\dfrac{**}{2\sqrt{2}+6}\times\dfrac{2\sqrt{2}-6}{2\sqrt{2}-6}$ if $\times\dfrac{\sqrt{2}}{\sqrt{2}}$ first ; then (Numerator =) $12\sqrt{2}-36$ **A1**
(Denominator = $8+12\sqrt{2}-12\sqrt{2}-36$) = –28 **B1** etc

If **A1 cso** is earned then condone incorrect *p* and *q* values stated.
If candidate has the correct answer and then "simplifies to" eg $9-3\sqrt{2}$ then withhold **A1** cso.

Q2	Solution	Mark	Total	Comment
(a)(i)	Gradient of $QR = -\frac{7}{5}$ **OE**	B1	1	do not penalise incorrect rearrangement of equation if correct gradient is stated
(ii)	Gradient of perp to $QR = \frac{5}{7}$	M1		**FT** negative reciprocal of their **(a)(i)**
	$(y-3)=\frac{5}{7}(x+2)$ **or** $y=\frac{5}{7}x+c, c=\frac{31}{7}$	A1		**ACF** with – – simplified to + etc
	$5x-7y+31=0$	A1	3	integer coefficients – all terms on one side. eg $0=7y-5x-31$
(b)	$7x+5y-2=0$ & $5x-3y+15=0$ eg $25x+75+21x-6=0$	M1		**correct** equations used and **correct** elimination of x or y eg $46x+69=0$ or $46y-115=0$ etc
	$x=-\frac{3}{2}$ **or** $x=-\frac{69}{46}$ **or** $y=\frac{5}{2}$ **or** $y=\frac{115}{46}$	A1		either x or y correct in any equivalent form
	{both $x=-\frac{3}{2}$ **and** $y=\frac{5}{2}$ **}** **or** $(-1.5, 2.5)$	A1	3	both coordinates written in simplest form (fractions or decimals) eg $\left(-1\frac{1}{2}, 2\frac{1}{2}\right)$
(c)	$(k+3--2)^2+(5-k-3)^2=...$	M1		$(k+5)^2+(2-k)^2=...$ may be under square root
	$2k^2+6k-140\ (=0)$	A1		or $k^2+3k-70\ (=0)$
	$(2)(k+10)(k-7)$	A1		**correct** factors or **correct** use of formula as far as $\frac{-6\pm\sqrt{1156}}{4}$ **OE** (or completing square).
	$k=-10,\quad 7$ **OE**	A1	4	
	Total		11	

(a)(i) **B0** for "$-\frac{7}{5}x$" but may earn **M1** in **(a)(ii)** if recovers.

(b) $7x+5\left(\frac{5}{3}x+5\right)-2=0$ earns **M1,** however $7x+5\left(\frac{5}{3}x-5\right)-2=0$, for example, scores **M0.**

Other examples scoring **M1** are $5\left(\frac{2}{7}-\frac{5y}{7}\right)-3y+15=0$; $\frac{5}{3}x+5=\frac{2}{5}-\frac{7}{5}x$

Accept any correct equivalent fraction for first **A1** but must have **both** $x=-\frac{3}{2}$ **and** $y=\frac{5}{2}$ for final **A1**.

NMS $\left(-1\frac{1}{2}, 2\frac{1}{2}\right)$ scores **M1 A1 A1**

(c) Poor use of or missing brackets resulting in **correct quadratic** can score the **M1** by implication; otherwise it scores **M0**

If completing square must reach form equivalent to $(k+1.5)=\pm\sqrt{\frac{289}{4}}$ for **second A1**

Q3	Solution	Mark	Total	Comment
(a)(i)	$\left[\text{p}(-2)=\right]\quad (-2)^3-7(-2)^2-5(-2)+26$	M1		clear attempt at p(–2) NOT long division
	$=-8-28+10+26$ $=0$ therefore $x+2$ is a factor	A1	2	must see powers of –2 simplified correctly working showing that p(–2)=0 and correct statement
(ii)	$b=-9$ **or** $c=13$	M1		by inspection
	$\left[\text{p}(x)=\right]\quad (x+2)(x^2-9x+13)$	A1	2	correct product with brackets correct
(b)(i)	b^2-4ac for "their" quadratic as far as $\left[(-9)^2-4\times13=\right]\quad 81–52$	M1		condone -9^2 if recovered as 81
	$29>0$ **or** $81–52>0$ **or (*)** (so curve crosses x-axis) 3 times	A1	2	**(*)** stating quadratic has 2 (real) roots correct deduction and quadratic correct
(ii)	$\left[\frac{\text{d}y}{\text{d}x}=\right]\quad 3x^2-14x-5$	M1 A1		2 terms correct all correct
	$\left[\frac{\text{d}^2y}{\text{d}x^2}=\right]\quad 6x-14$	B1	3	
(iii)	$\left[\frac{\text{d}y}{\text{d}x}=\right]\quad 3\left(-\frac{1}{3}\right)^2-14\left(-\frac{1}{3}\right)-5$ **or** $\left[\frac{\text{d}^2y}{\text{d}x^2}=\right]\quad 6\left(-\frac{1}{3}\right)-14$	M1		correct substitution of $x=-\frac{1}{3}$ into "their" $\frac{\text{d}y}{\text{d}x}$ **or** "their" $\frac{\text{d}^2y}{\text{d}x^2}$
	$\frac{\text{d}y}{\text{d}x}=\frac{1}{3}+\frac{14}{3}-5=0$	A1		convincingly showing $\frac{\text{d}y}{\text{d}x}=0$ **and** $\frac{\text{d}y}{\text{d}x}=\ldots$ **must** appear on at least one line
	$\frac{\text{d}^2y}{\text{d}x^2}=-2-14(=-16)<0$ $\Rightarrow$ maximum	A1	3	correct and $\frac{\text{d}^2y}{\text{d}x^2}$ seen & value shown to be < 0 & statement **must** earn **M1 A1** to earn final **A1**
	Total		12	

(a)(i)	Minimum required for statement is "$\therefore$ factor" Powers of –2 must be evaluated: **Example "**p(–2) $=-8-28+10+26$ $=0$ so factor" scores **M1 A1** Statement may appear first : **Example "** x+2 is factor if p(–2) = 0 & p(–2) $=-8-28+10+26=0$" scores **M1 A1** but **Example "** $\text{p}(-2)=(-2)^3-7(-2)^2+5(-2)+26$ $=0$ therefore x+2 is a factor" scores **M1 A0**
(ii)	**M1** may also be earned for a full long division attempt by (x+2), or a clear attempt to find a value for both b and c (even though incorrect) by comparing coefficients. **NMS** $(x+2)(x^2-9x+13)$ scores **M1A1**
(b)(i)	Do not penalise $9^2-4\times13$ etc for **M1 ;** may use full quadratic equation formula so award **M1** for correct unsimplified expression with discrim't as far as "81–52" for "their" quadratic. **NMS** "3 times" scores **M0.**
(iii)	May show $3x^2-14x-5=(3x+1)(x-5)$ **M1** with $\frac{\text{d}y}{\text{d}x}=0$ leading to $x=-\frac{1}{3}$ for first **A1**. Withhold final **A1** if incorrect statement such as "therefore maximum" follows $\frac{\text{d}y}{\text{d}x}=0$

Q4	Solution	Mark	Total	Comment
	$b^2 - 4ac = 0$	B1		condition for equal roots stated **or** correct discriminant = 0
	$(5k-3)^2 - 4 \times 3k(k+1) \quad (=0)$	M1		correct discriminant
	$13k^2 - 42k + 9\ (=0)$	A1		
	$(k-3)(13k-3)\ (=0)$	dM1		attempt at factors or **correct substitution** into formula for their quadratic
	$k = 3, \quad k = \frac{3}{13}$	A1cso	5	accept equivalent fractions
	Total		5	

Condone poor use/omission of brackets for **M1** if correct discriminant is intended, but the **A1 cso** cannot then be earned even if recovered later.

For **dM1** factors must be such that the product would give "their" k^2 and constant terms; if quadratic formula is used then it must be a **correct substitution** for "their" quadratic.

Candidates must have "= 0" on at least one line of working or statement "$b^2 - 4ac = 0$" and all working correct to earn **A1cso.**

If candidate uses " > 0" etc then withhold **A1cso** even if final answer is written as $k = 3, \quad k = \frac{3}{13}$.

M1 only if discriminant within formula

Q5	Solution	Mark	Total	Comment
(a)	Grad $PC = \dfrac{-2--8}{2-7}$	**M1**		condone one sign error in one term
	$= -\dfrac{6}{5}$ **OE**	**A1**	2	withhold **A1** if gradient of perpendicular attempted. No **ISW** here.
(b)	$(x-7)^2+(y+8)^2$ **=…**	**M1**		**or** $(x-7)^2+(y--8)^2$ **=…**
	5^2+6^2 **or** $25+36$ **or** 61	**B1**		**or** seen under square root
	$(x-7)^2+(y+8)^2=61$	**A1**	3	**or** $(x-7)^2+(y--8)^2=61$
(c)	$-8+\text{"}their\text{"}\sqrt{k}$ **or** $-8\pm\text{"}their\text{"}\sqrt{k}$	**M1**		also allow $-8-\text{"}their\text{"}\sqrt{k}$ for **M1**
	$-8+\sqrt{61}$	**A1**	2	
(d)	*M* is midpoint of *PR*			
	$(CM^2=)$ "*their* 61"-4^2	**M1**		Pythagoras used correctly with "4" and with hyp^2 = "*their*" *k* or correct
	$(CM^2=)$ 45	**A1**		or $(CM=)$ $\sqrt{45}$
	(shortest distance $=$) $3\sqrt{5}$	**A1cso**	3	all notation correct
	Total		**10**	

(a) Award **SC B1** for grad $PC = 6/5$ if **M1** not earned

NMS $-\dfrac{6}{5}$ scores **M1 A1 ;** condone $\dfrac{-6}{5}$ **or** $\dfrac{6}{-5}$ for full marks

(b) $(x-7)^2+(y+8)^2=61$ scores **M1 B1 A1**

allow RHS = $\left(\sqrt{61}\right)^2$ instead of 61 for full marks

Example: $(x-7)^2+(y+8)^2=\sqrt{61}$ earns **M1 B1 A0**

Equation of circle must be written explicitly as $(x-7)^2+(y+8)^2=61$
or $(x-7)^2+(y--8)^2=61$ to earn **A1** mark

(c) **NMS** $-8+\sqrt{61}$ scores **M1 A1**

Alternative: $y^2+16y+3=0 \Rightarrow y=\dfrac{-16\pm\sqrt{256-12}}{2}$ **M1** $\Rightarrow y=\dfrac{-16+\sqrt{244}}{2}$ **A1**

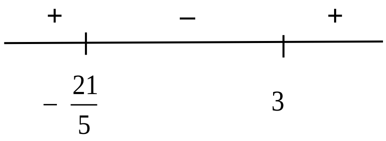
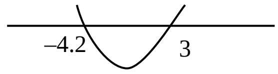
(d) **Example:** $61-4^2=45=3\sqrt{5}$ scores **M1, A1, A0**

Example: $61-4^2=45,\quad \sqrt{45}=3\sqrt{5}$ scores **M1, A1, A1**

Q6	Solution	Mark	Total	Comment
(a)(i)	$\left[\frac{dy}{dx}=\right] \quad 9x^2-7$	M1 A1		one term correct all correct (no +c etc)
	when $x=-1$, $\frac{dy}{dx}=(9-7=)\,2$	A1		
	$y-14=$"*their* 2"$(x--1)$	dM1		or $y=$"*their* 2"$x+c$ & attempt to find c using $x=-1$ and $y=14$
	$y=2x+16$; $y-14=2(x+1)$ **OE**	A1	5	**ACF**
(ii)	$[y=0] \quad x=-8$	B1	1	must have correct **(a)(i)**
(b)(i)	$\frac{3x^4}{4}-\frac{7x^2}{2}+10x\,(+c)$	M1 A1		two terms correct all correct
	$\left[\frac{3\times(-1)^4}{4}-\frac{7\times(-1)^2}{2}+10\times(-1)\right]-\left[\frac{3\times(-2)^4}{4}-\frac{7\times(-2)^2}{2}+10\times(-2)\right]$	dM1		“their” F(–1) – F(–2)
	$\left[\frac{3}{4}-\frac{7}{2}-10\right]-\left[\frac{48}{4}-\frac{28}{2}-20\right]$	A1		**correct** with powers of (–1) , (–2) and minus signs handled correctly (– – = +)
	$=9\tfrac{1}{4}$	A1	5	9.25 , $\frac{37}{4}$, $\frac{111}{12}$ **OE** or correct **single** equivalent fraction
(ii)	Area of triangle $=\left(\tfrac{1}{2}\times14\times7=\right)49$	B1		allow $\frac{196}{4}$ etc
	Region area = “their” Δ –“their”**(b)(i)**	M1		“their” ($49-9\tfrac{1}{4}$)
	$=39\tfrac{3}{4}$	A1	3	39.75 , $\frac{159}{4}$, **OE**
	Total		**14**	

(a)(i)	Must simplify “– – “ to “+” not simply $y-14=2(x--1)$ for final **A1**;
(b)(i)	Must combine terms for final **A1; Example …** $3\tfrac{1}{4}+6$ scores final **A0.**
(ii)	May find triangle area by integration for **B1**. For **M1** condone use of “their”**(b)(i)** – “their” Δ if appropriate for their values. Be generous in awarding this **M1** provided they are considering the area of a triangle.

Q7	Solution	Mark	Total	Comment
(a)(i)	$2\left(x-\frac{5}{4}\right)^2 \ldots$	**M1**		$2(x-1.25)^2$ **… OE**
	$2\left(x-\frac{5}{4}\right)^2+\frac{7}{8}$	**A1**	**2**	$2(x-1.25)^2+0.875$ **OE**
(ii)	$x=\frac{5}{4}$ $(x=1.25)$ **OE**	**B1F**	**1**	**FT** their $x=p$
(iii)	$y=\frac{7}{8}$ $(y=0.875)$ **OE**	**B1F**	**1**	**FT** their $y=q$ or $y-q=0(x-p)$ etc
(b)	$2(x-3)^2-5(x-3)\ldots$	**M1**		or "their" $2\left(x-\frac{5}{4}-3\right)^2 \ldots$
	$(y=)$"their" $\text{f}(x)$ -8	**B1**		or $y+8=$"their" $\text{f}(x)$
				for guidance $y=2\left(x-\frac{17}{4}\right)^2-\frac{57}{8}$
	$a=-17$	**A1**		OE such as $-\frac{68}{4}$
	$b=29$	**A1**	**4**	OE such as $\frac{232}{8}$
	Total		**8**	
(a)(i)	If **M1** is not earned, award **SC1** for $2\left(x-\frac{5}{4}\right)+\frac{7}{8}$ If comparing coefficients and **M1** is not earned, then award **SC1** for $p=\frac{5}{4}$ $q=\frac{7}{8}$			
(ii)	Must have $\boldsymbol{x=}$"*their*"p for **B1F** and strict follow through			
(iii)	Must have $\boldsymbol{y=}$"*their*"q for **B1F** and strict follow through			
(b)	Full marks for $y=2x^2-17x+29$ **-** not required to write $a=-17$**,** $b=29$			

Q8	Solution	Mark	Total	Comment
	$5x^2+6x-63<0$			
(a)	$(5x+21)(x-3)$	**M1**		correct factors or correct use of formula as far as $\frac{-6\pm\sqrt{1296}}{10}$ or completing square as far as $-\frac{3}{5}\pm\sqrt{\frac{324}{25}}$
	CVs are $x=3,\ -\frac{21}{5}$	**A1**		condone equivalent fractions here
		**M1**		use of sign diagram or graph; **PI** by correct answer 
	$-\frac{21}{5}<x<3$ **or** $3>x>-4.2$ **etc**	**A1**	**4**	fractions must be simplified for final mark; no **ISW** here
(b)(i)	$2x(x+3+4x+3)<126$ $10x^2+12x<126 \Rightarrow 5x^2+6x<63$	**B1**	**1**	$4x(x+3)+6x^2<126$ etc **AG** be convinced; condone trailing equals sign and final answer as $63>5x^2+6x$
(ii)	AD =5x so perimeter = $14x+6$	**B1**	**1**	condone $6+14x$
(iii)	"their" $14x+6\ \geq 30$	**M1**		must have "greater than or equal to"
	$x \geq \frac{12}{7}$	**A1**		condone $x \geq \frac{24}{14}$
	combining gives $\frac{12}{7}\leq x<3$	**A1**	**3**	condone $\frac{24}{14}\leq x<3$ must have scored 4 marks in part **(a)**
	Total		**9**	

(a) For second **M1**, if critical values are correct then sign diagram or sketch must be correct ***with correct CVs marked***.
However, if CVs are not correct then second **M1** can be earned for attempt at sketch or sign diagram but ***their CVs*** MUST be marked on the diagram or sketch.
Final **A1,** inequality must have x and no other letter.

Final answer of $x>-\frac{21}{5}$ AND $x<3$ (with or without working) scores **4 marks**.

(A) $-\frac{21}{5}<k<3$ (B) $x>-\frac{21}{5}$ OR $x<3$ (C) $x>-\frac{21}{5}$, $x<3$ (D) $-\frac{21}{5}\leq x\leq 3$
with or without working, each scores **SC3**

Example NMS $\frac{21}{5}<x<3$ scores **M0 (**since one CV is incorrect)

Example NMS $x<-4.2 \quad x<3$ scores **M1 A1 M0 (**since both CVs are correct)

(b)(iii) If **M1** is not earned award **SC B1** for $\frac{12}{7}<x<3$