AQA

Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

AS MATHEMATICS

Unit Pure Core 1

Wednesday 17 May 2017 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

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Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Express $\dfrac{1+4\sqrt{7}}{5+2\sqrt{7}}$ in the form $m + n\sqrt{7}$, where m and n are integers.

[4 marks]

(b) Solve the equation

$$x\left(9\sqrt{5}-2\sqrt{45}\right)=\sqrt{80}$$

giving your answer in its simplest form.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 1

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QUESTION PART REFERENCE	Answer space for question 1

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2 A curve has equation $y = 20x - x^2 - 2x^3$. The curve has a stationary point at the point M where $x = -2$.

(a) Find the x-coordinate of the other stationary point of the curve.

[4 marks]

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point M, and hence determine, with a reason, whether M is a minimum point or a maximum point.

[3 marks]

(c) Sketch the curve.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE

Answer space for question 2

Turn over ►

3 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + bx^2 + cx + 24$$

where b and c are integers.

(a) Given that $x+2$ is a factor of $\mathrm{p}(x)$, show that $2b - c + 8 = 0$.

[2 marks]

(b) The remainder when $\mathrm{p}(x)$ is divided by $x-3$ is -30.

Obtain a further equation in b and c.

[2 marks]

(c) Use the equations from parts **(a)** and **(b)** to find the value of b and the value of c.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 3**

QUESTION PART REFERENCE	Answer space for question 3

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4 The point A has coordinates $(-2, 5)$ and the point B has coordinates $(8, -6)$.

(a) Find an equation for the straight line AB, giving your answer in the form $px + qy = r$, where p, q and r are integers.

[4 marks]

(b) The point C has coordinates $(k, k+1)$. Given that angle ACB is a right angle, find the two possible values of k.

[5 marks]

QUESTION PART REFERENCE	Answer space for question 4

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QUESTION PART REFERENCE	Answer space for question 4

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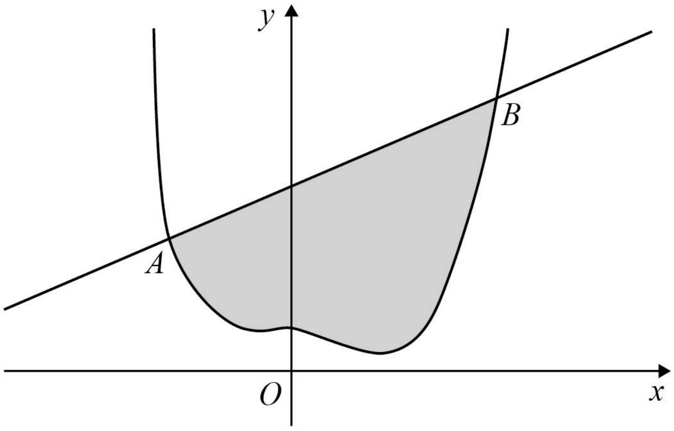
5 A curve and the line AB are sketched below.



The curve has equation $y = 2x^4 - 3x^3 + 4$ and the points $A(-1, 9)$ and $B(2, 12)$ lie on the curve.

(a) Find the equation for the normal to the curve at the point A, giving your answer in the form $y = mx + c$.

[5 marks]

(b) (i) Find $\int_{-1}^{2} \left(2x^4 - 3x^3 + 4\right) \mathrm{d}x$.

[5 marks]

(ii) Hence find the area of the shaded region bounded by the curve and the line AB.

[3 marks]

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6 A circle with centre C has equation $x^2 + y^2 + 20x - 14y + 49 = 0$.

(a) Express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$

[3 marks]

(b) Show that the circle touches the y-axis and crosses the x-axis in two distinct points.

[4 marks]

(c) A line has equation $y = kx + 2$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the circle and the line satisfy the equation

$$(1+k^2)x^2 + 10(2-k)x + 25 = 0$$

[2 marks]

(ii) Hence, find the value of k for which the line is a tangent to the circle.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 6

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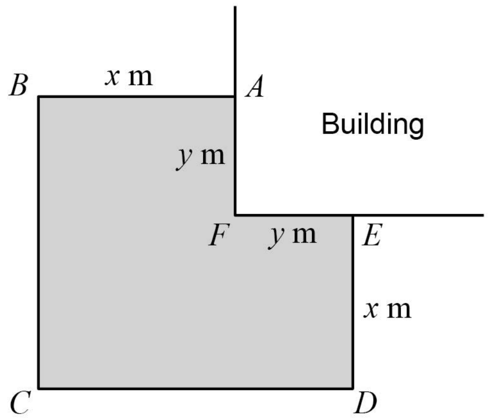
7 The diagram shows the right-angled corner AFE of a building and four sections of fencing running parallel to the walls of the building.



Each of the sections of fencing AB and DE has length x metres and each of the sections of wall AF and FE has length y metres. The total length of the four sections of fencing AB, BC, CD and DE is 15 metres. The shaded region bounded by the fencing and the walls of the building has area $S\,\text{m}^2$.

(a) (i) Express y in terms of x.

[2 marks]

(ii) Show that $S = 3(5x - x^2)$.

[3 marks]

(b) (i) Express $5x - x^2$ in the form $p - (x - q)^2$, where p and q are rational numbers.

[2 marks]

(ii) Hence find the maximum value of S.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 7

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QUESTION PART REFERENCE	**Answer space for question 7**

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8 The water level in a reservoir rises and falls during a four-hour period of heavy rainfall. The height, h cm, of water above its normal level, t hours after it starts to rain, can be modelled by the equation

$$h = 4t^3 - \frac{59}{2}t^2 + 72t, \quad 0 \leqslant t \leqslant 4$$

(a) Find the rate of change of the height of water, in cm per hour, 3 hours after it starts to rain.

[4 marks]

(b) Find the values of t for which the height of the water is decreasing.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 8**

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END OF QUESTIONS