AS

Mathematics

MPC2-Pure Core 2

Mark scheme

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1(a)	$h = 0.25$	**B1**		$h = 0.25$ OE stated or used (PI by x-values 0, 0.25, 0.5, 0.75 OE provided no contradiction)
	$f(x) = \sqrt{9-16x^3}$ $I \approx \frac{h}{2}\{f(0)+f(3/4)+2[f(1/4)+f(1/2)]\}$	**M1**		$h/2\{f(0)+f(3/4)+2[f(1/4)+f(1/2)]\}$ OE summing of areas of the 'trapezia'.. **M0** if using an incorrect f(x)
	$\frac{h}{2}$ with {}= $\sqrt{9}+\sqrt{2.25}+2\left(\sqrt{8.75}+\sqrt{7}\right)$	**A1**		OE Accept 3sf or better evidence for surds Can be implied by later <u>correct</u> work provided >1 term or a single term for I which rounds to 1.96
	$=\frac{h}{2}\{ 3 + 1.5 +2[2.95(8..) + 2.64(5..)]\}$ $=\frac{h}{2}\{ 4.5 + 11.2(07..)]\}$ ($I \approx 0.125\times15.7(07..)$) (= 1.96(3…) I = 1.96 (to 3 sf)	**A1**	**4**	CAO Must be 1.96 **SC** 4 strips used: **Max B0M1A0**, 1.98 **A1**
(b)	Increase the number of ordinates	**E1**	**1**	OE eg Increase the number of strips.
	Total		**5**	
	NO MISREADS ALLOWED IN THIS QUESTION In the M1 line if brackets missing look for later evidence of correct method before awarding the M1 $\frac{\sqrt{35}}{2}$ is a common OE for $\sqrt{8.75}$ For guidance, separate trapezia, 0.744(7..) + 0.700(4..)+0.518(2..)			

Q	Solution	Mark	Total	Comment
2(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=3+\frac{3}{2}x^{\frac{1}{2}}$	**B2,1**	**2**	ACF. If not **B2**, award **B1** for correct differentiation of either $x^{3/2}$ or $3x-7$
(b)(i)	$(k=)\ 13$	**B1**	**1**	13
(ii)	At $P(4,k)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3+\frac{3}{2}(4)^{0.5}$ (= 6)	**M1**		Attempt to find c's $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 4$. M0 if c's answer (a) is a constant
	Gradient of normal = $-\frac{1}{6}$	**dM1**		$m \times m' = -1$ used
	Eqn of normal $y-13=-\frac{1}{6}(x-4)$	**A1F**	**3**	ACF **only** ft on c's non-zero value of k ie check c's equation is equivalent to $6y+x=4+6k$, for c's non-zero value of k
(iii)	When $y = 0$, $0-13=-\frac{1}{6}(x-4)$	**M1**		Attempts to find x when y=0 in c's **linear** equation answer to (b)(ii)
	$x=82$	**A1**	**2**	82 82 with or without working scores 2/2
	Total		**8**	
(b)(i)	Condone 'y=13' unless there is any contradiction to 13 being the final value			

Q	Solution	Mark	Total	Comment
3(a)	$(AC^2 =)\ 6^2 + 10^2 - 2(6)(10)\cos\frac{2\pi}{3}$	**M1**		RHS of cosine rule used correctly.
	$AC^2 = 36 + 100 + 60 \quad (= 196)$	**A1**		PI by next line
	$AC = 14$ (cm)	**A1**	**3**	CAO 14
(b)(i)	Arc $AB = r\theta$	**M1**		$r\theta$ used for arc length with $\theta = \frac{4\pi}{3}$ or $\frac{2\pi}{3}$ or $\frac{\pi}{3}$ OE. Accept equivalent work in degrees form
	$= 6 \times \left(2\pi - \frac{2\pi}{3}\right)$	**A1**		Correct numerical expression which would simplify to 8π eg $6\pi + 6\frac{\pi}{3}$, $12\pi \times \frac{2}{3}$ or 25.13 or an AWRT 25.13
	= 25.13…. = 25.1 (cm) (to 3sf)	**A1**	**3**	AG Must see at least a correct 4sf value before the final printed answer 25.1 is stated.
(ii)	(Area of triangle=) $0.5 \times 6 \times 10 \sin\left(\frac{2\pi}{3}\right)$	**M1**		A correct numerical expression for the area of the triangle. PI by expression simplifying to $\sqrt{675}$ or PI by value 25.9.. or 26
	(Area of sector=) $0.5 \times 6^2 \times \left(2\pi - \frac{2\pi}{3}\right)$	**M1**		A correct numerical expression for the area of the sector, which would simplify to 24π PI by value 75 or 75.4 or 75.3…
	(Area of triangle =) $15\sqrt{3} = 25.9…$. (Area of sector=) 24π	**A1**		Dep on just the relevant one M1 for a correct value for **either** area of triangle ($\sqrt{675}$ OE simplified or 26 or 25.9 or 25.9…) **or** area of sector (24π or 75.4 or 75 or 75.3…). PI by either $\sqrt{675} + 24\pi$ OE simplified or final AWRT 101
	(Area of region=) 26 + 75 = 101 (cm^2)	**A1**	**4**	CAO 101
	Total		**10**	

(a)	$AC^2 = 36 + 100 - - 60$ look for later evidence before awarding the 1st A1 mark
(b)(i)	$6 \times \frac{2\pi}{3} = 12.56...$ $12.566... \times 2 = 25.13.. = 25.1$ (to 3sf) without any justification for x2 scores M1A0A0
(b)(i)	…..=8π = 25.1 does **NOT** score the final A1; …..=$8\pi = 25.1\cancel{3}$ **GETS** the final A1.
(b)(ii)	Area of triangle = $0.5 \times 6 \times 10 \sin\left(\frac{\pi}{3}\right)$ is **M0 unless** it is supported by a statement that eg sin120=sin60

Q	Solution	Mark	Total	Comment
4(a)(i)	$(d=)\ 9$	**B1**	**1**	9
(ii)	$(u_{100}=)\ 23+(100-1)d$	**M1**		$23+(100-1)d$ OE; ft on c's answer for part **(a)(i)** if d is numerical
	$(u_{100}=)\ 914$	**A1**	**2**	914 **NMS** 914 scores 2/2
(iii)	Number of terms, $N = 201$	**B1**		$N = 201$ stated or used
	$(\sum_{n=100}^{300} u_n) = \frac{201}{2}[914+2714]$	**M1**		Either $\frac{201}{2}[u_{100}+2714]$ or $\frac{201}{2}[2u_{100}+(201-1)d]$ used with c's values for d and u_{100} from parts **(a)(i)** and **(a)(ii)** .
	$\sum_{n=100}^{300} u_n = 364614$	**A1**	**3**	364614 **NMS** 364614 scores 3/3 **SC** If 0/3 scored then award 1 mark if 363700 is the c's final answer
(a)(iii) ALTn	$\sum_{n=100}^{300} u_n = \sum_{n=1}^{300} u_n - \sum_{n=1}^{99} u_n$	**(B1)**		$\sum_{n=1}^{300}(u_n) - \sum_{n=1}^{99}(u_n)$ OE eg $S_{300}-S_{99}$
	$= \frac{300}{2}[23+2714] - \frac{99}{2}[23+u_{100}-d]$ $(= 410550 - 45936)$	**(M1)**		OE Ft c's values for d and u_{100} from parts **(a)(i)** and **(a)(ii)**
	$\sum_{n=100}^{300} u_n = 364614$	**(A1)**	**(3)**	364614 **SC** If 0/3 scored then award 1 mark if 363700 is the c's final answer
(b)(i)	$24+24r^3\ (=-57)$	**M1**		$a+ar^3$ seen or used with $a=24$. PI by (r=)-1.5 oe
	$r=-\frac{3}{2}$	**A1**	**2**	Correct value for common ratio.
(ii)	No, (the series does NOT have a sum to infinity), since -1.5 does not lie between -1 and 1	**E1ft**	**1**	OE statement with no contradiction Conclusion and reason, ft only on c's value of r being <-1. Do **NOT** apply ISW
	Total		**9**	

(a)(iii)	Accept eg notation $\sum_{n=1}^{300} - \sum_{n=1}^{99}$ for **(B1)** OE for the (M1) in Altn: $\frac{300}{2}[2(23)+299d] - \frac{99}{2}[2(23)+98d]$
(b)(ii)	Any value of $r \geq -1$ in (b)(i) scores E0 in (b)(ii). For any value of $r<-1$ in (b)(i), 'No, as $r<-1$'; 'No, as $r \leq -1$'; 'No, as $\lvert r \rvert > 1$'; 'No, as $\lvert r \rvert \geq 1$'; are examples where E1ft is scored.

Q	Solution	Mark	Total	Comment
5(a)	$a = 54;\quad b = 36;\quad c = 8$	**B1,B1, B1**	**3**	**B1** for each correct value. Accept correct embedded values for a, b and c within the expansion linked to correct power of x
(b)	$(n =) -4$	**B1**	**1**	-4 . Condone x^{-4}.
(c)(i)	(Integrand =) $x^{-4}\left(27 + ax^2 + bx^4 + cx^6\right)$	**M1**		Uses c's **(a)** and c's **(b)** in a product; **or** cancelling to get at least 3 correct ft terms $\frac{27}{x^4} + \frac{a}{x^2} + b + cx^2$ (allow bx^0 for b)
	$\int \left(27x^{-4} + ax^{-2} + b + cx^2\right)$ (dx)	**A1F**		Integrand $27x^{-4} + ax^{-2} + b + cx^2$, only ft on c's non-zero numerical answers from **(a)**. PI by next line in soln.
	$= \frac{27x^{-3}}{-3} + \frac{ax^{-1}}{-1} + bx + \frac{cx^3}{3}$ $(+k)$	**dM1**		Correct integration of at least 3 terms, ft only on c's non-zero values from **(a)**, accept unsimplified
	$= -9x^{-3} - 54x^{-1} + 36x + \frac{8x^3}{3}$ $(+k)$	**A1**	**4**	Correct with coefficients and signs simplified; condone absence of $+k$
(c)(ii)	$\left\{-9(3)^{-3} - 54(3)^{-1} + 36(3) + \frac{8(3)^3}{3}\right\} -$ $\left\{-9 - 54 + 36 + \frac{8}{3}\right\}$	**M1**		Clear evidence of F(3) – F(1) attempted where integration must have been attempted to get F; If cand uses F(x)= the given integrand, then **M0**
	$= \left[-\frac{1}{3} - 18 + 108 + 72\right] - \left(-24\frac{1}{3}\right)$			
	= 186	**A1**	**2**	CAO 186 NMS scores 0/2
	Total		**10**	

(c)(i)	The final A1 mark can be awarded if the simplified version occurs in (c)(ii) before the values are inserted
(c)(ii)	For guidance $\frac{485}{3} - \left(-\frac{73}{3}\right) = \frac{558}{3}$ = 186
(c)(ii)	After incorrect integration we must see at least some substitution of 3 and 1 rather than just the difference of two incorrect values for M1 to be awarded

Q	Solution	Mark	Total	Comment
6	$100 = 121p + q$	**M1**		OE seen or used
	$16 = 16p + q$	**M1**		OE seen or used
	$100 = 121p + 16 - 16p \quad (84 = 105p)$	**dM1**		Valid method to solve the correct two simultaneous equations to reach a correct equation in either p only or q only eg $100 = 121\left(1 - \frac{q}{16}\right) + q$ OE, $21 = \frac{105}{16}q$, $16(121 - 100) = (121 - 16)q$ PI by correct values for both p and q
	$p = \frac{84}{105} \quad \left(= \frac{4}{5} = 0.8\right)$ $q = \frac{336}{105} \quad \left(= \frac{16}{5} = 3.2\right)$	**A1**		A correct value for **both** p and q. ACF
	$u_4 = 0.8 \times 100 + 3.2 = 83.2 \quad \left(= \frac{416}{5}\right)$	**A1F**	**5**	FT provided either c's p or c's q is correct ie ft on either $80 + q$ or $100p + 3.2$ Accept correct ft value in any form
	Total		**5**	

Q	Solution	Mark	Total	Comment
7(a) (i)	$\log_b \dfrac{6x}{18}$	B1	1	$\log \dfrac{6x}{18}$ OE Condone base b missing
(ii)	$\log_b \dfrac{6x}{18} + \log_b(x-1) = \log_b \dfrac{6x(x-1)}{18}$	M1		Eg $\log D + \log(x-1) = \log D(x-1)$ or $\log(x+4) - \log(x-1) = \log\left(\dfrac{x+4}{x-1}\right)$ OE results so as to have no more than two log terms remaining in the given equation. Condone base b missing PI by a correct eqn. with no log terms provided no errors seen in (ii) in determining such an eqn.
	$\log_b(x+4) = \log_b \dfrac{6x(x-1)}{18}$ $\Rightarrow x+4 = \dfrac{6x(x-1)}{18}$	A1		OE A correct eqn after all logarithms eliminated in a correct manner. Condone $\log(x+4) = \log \dfrac{6x(x-1)}{18}$ with 'log' on each side crossed out.
	$x^2 - 4x - 12 = 0 \Rightarrow x = 6, \quad x = -2$	A1		$x = 6, \quad x = -2$; if -2 is missing we must see either $(x-6)(x+2)$ or a valid statement for ignoring it
	$x = 6$	A1	4	6 as the only solution.
(b)(i)	$n = m^k$	B1	1	
(ii)	$(\log_2)x^2\sqrt{x} = (\log_2)x^{2.5}$	B1		$x^2\sqrt{x} = x^{2.5}$ seen or used at any stage
	$p\log_8 x^2 = \log_8(x^2)^p$	M1		Use of log law $a\log b = \log b^a$ at any stage in **(b)(ii)**, or $8^T = x^{2p}$ OE seen
	Let $T = p\log_8 x^2 = \log_2 x^2\sqrt{x}$ eg $2^T = x^{2.5}$, $8^T = x^{2p} = 2^{3T}$; $\underline{x^{2p} = x^{7.5}}$ eg $\log_2 x^{2.5} = \log_8 x^{2p} = \dfrac{\log_2 x^{2p}}{\log_2 8}$ $= \log_2 x^{\frac{2p}{3}}$; $\underline{x^{2.5} = x^{\frac{2p}{3}}}$	M1		Correctly converting to the same base OE and eliminating in a correct manner all logarithmic terms. Can also be awarded after B0 if cand has $x^2\sqrt{x} = x^q$, where q is non-integer
	$2p = 7.5$; $p = 3.75$ or eg $\dfrac{2p}{3} = 2.5$; $p = 3.75$	A1	4	Correct value for p, obtained convincingly for a general x. NMS scores 0/4
	Total		**10**	
(a)(ii)	Example: RHS of given eqn = $\log 6x - \log 18(x-1)$ scores M0			
(b)(ii)	$2p\log_8 x = 2\log_2 x + 0.5\log_2 x$ **(M1B1)**; $= \dfrac{2}{3}p\log_2 x$ **;** $2.5 = \dfrac{2p}{3}$ **(M1)** $p = 3.75$ **(A1)**			

Q	Solution	Mark	Total	Comment
8(a)	$9\sin^2\theta - 2\sin\theta\cos\theta = 8$ $\dfrac{9\sin^2\theta}{\cos^2\theta} - \dfrac{2\sin\theta\cos\theta}{\cos^2\theta} = \dfrac{8}{\cos^2\theta}$ $9\tan^2\theta - 2\tan\theta = \dfrac{8}{\cos^2\theta}$	**M1**		Dividing each term by $\cos^2\theta$ and using correct identity to obtain at least two correct terms in different powers of $\tan\theta$
	.... $= 8(\cos^2\theta + \sin^2\theta)$	**M1**		Replacing 8 by $8(\cos^2\theta + \sin^2\theta)$ or PI by seeing eg $\sin^2\theta - 2\sin\theta\cos\theta =$ $= 8(1-\sin^2\theta) = 8\cos^2\theta$ or PI by seeing $\dfrac{8}{\cos^2\theta} = 8(1+\tan^2\theta)$ [**The two method marks can be awarded in any order**]
	$9\tan^2\theta - 2\tan\theta = \dfrac{8(\cos^2\theta + \sin^2\theta)}{\cos^2\theta}$ $9\tan^2\theta - 2\tan\theta = 8 + 8\tan^2\theta$ $\tan^2\theta - 2\tan\theta - 8 = 0$ $(\tan\theta - 4)(\tan\theta + 2) = 0$	**A1**	**3**	AG Be convinced
(b)	$75.96,\ 255.96,\ 116.56,\ 296.56$	**M1**		Any **two** correct values equal to or rounding to integer values 76, 256, 117, 297 seen
	($\tan 2x = 4$) $2x = 75.96,\ 255.96$ ($\tan 2x = -2$) $2x = 116.56,\ 296.56$	**A1**		$2x$ equal to or rounding to the four integer values 76, 256, 117, 297 **seen or used** Condone eg 2θ for $2x$
	($x =$) $38^\circ,\ 128^\circ,\ 58^\circ,\ 148^\circ$	**B2,1,0**	**4**	38, 58, 128, 148 with or without working scores **B2**; if **B2** not scored, award **B1** if either four values rounding to the above or three of the above and no more than one incorrect; or 38,128, 59, 149 [Ignore answers outside $0 \le x \le 180$. If more than four answers in interval deduct 1 mark for each extra from **B** marks to a min of 0]
	Total		**7**	

(a)	Line 2 followed by line 6 in the solution column would score **M1M1;** Line 2 then line 7 is **M1M0**
(a)	Cand who starts with $(\tan\theta - 4)(\tan\theta + 2) = 0$ and shows it is $9\sin^2\theta - 2\sin\theta\cos\theta = 8$ can score **M1M1**
(a)	If θ missing throughout the soln except for the printed result, do not award the A1 mark
(b)	Candidate who just solves the eqn given in (a) **without dividing solutions by 2**, can score **M1A0**
(b)	Gen. Solns: Either -63.4+180n, or 75.96+180n; (**M1**), [37.98+90n, -31.7+90n;] **A1**; 38,128,58,148 **B2**

Q	Solution	Mark	Total	Comment
9(a)(i)	Stretch (**I**) in x(-direction) OE (**II**) (scale factor) 0.5 OE (**III**)	**M1** **A1**	 **2**	Need (**I**) **and** either (**II**) or (**III**) Need (**I**) and (**II**) and (**III**) More than one transformation scores 0/2
(a)(ii)	Translation $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$	**E2,1**	**2**	**E2**: 'translat…' and $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$. If not **E2** award **E1** for either 'translat…' or for $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$. More than one transformation scores 0/2
(b)(i)	$2^{x+3} = 2^3 2^x = 8u$	**B1**	**1**	$8u$ Accept $2^3 u$ if in later work it is simplified to $8u$
(ii)	$u^2 - 8u + 15 = 0$ $y = \left(\dfrac{y+15}{2^3}\right)^2 \quad (y^2 - 34y + 225 = 0)$	**M1**		Eliminating y with $2^{2x} = u^2$ or $\left(2^x\right)^2$ to form a quadratic eqn in u or in 2^x (terms in any order) **OR** eliminating x with $2^{2x} = y = \left(\dfrac{y+15}{2^3}\right)^2$ condoning one sign or one numerical error
	$u = 3,\ u = 5$; $y = 9,\ y = 25$;	**A2,1**		Correct values for u (or 2^x) and correct values for y. If not **A2** award **A1** for any two correct values. If y-values not simplified look for later evidence; eg p as '16' in the final answer is sufficient evidence.
	$u = 3 \Rightarrow x = \log_2 3$; $u = 5 \Rightarrow x = \log_2 5$	**M1**		From a quadratic eqn, use of $2^x = k \Rightarrow x = \log_2 k$ OE, for k>0 **M0** if c's quadratic eqn would give non-real roots or no positive root when solved correctly
	Gradient of AB = $\dfrac{25-9}{\log_2 5 - \log_2 3}$	**dM1**		Dep on both previous M1. $\dfrac{y_A - y_B}{x_A - x_B}$ used with c's x and y values
	$= \dfrac{16}{\log_2\left(\dfrac{5}{3}\right)}$	**A1**	**6**	OE in the requested form eg $\dfrac{-16}{\log_2(0.6)}$
	Total		**11**	
(b)(ii)	OE for the 2nd M1: eg $2^x = 3 \Rightarrow x = \dfrac{\log 3}{\log 2}$			