Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MPC3

Unit Pure Core 3

Wednesday 19 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) Find $\frac{dy}{dx}$ when $y = (x^3 - 1)^6$. *(2 marks)*

(b) A curve has equation $y = x \ln x$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the tangent to the curve $y = x \ln x$ at the point on the curve where $x = e$. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

2 A curve is defined by the equation $y = (x^2 - 4)\ln(x + 2)$ for $x \geqslant 3$.

The curve intersects the line $y = 15$ at a single point, where $x = \alpha$.

(a) Show that α lies between 3.5 and 3.6. *(2 marks)*

(b) Show that the equation $(x^2 - 4)\ln(x + 2) = 15$ can be arranged into the form

$$x = \pm\sqrt{4 + \frac{15}{\ln(x + 2)}}$$ *(2 marks)*

(c) Use the iteration

$$x_{n+1} = \sqrt{4 + \frac{15}{\ln(x_n + 2)}}$$

with $x_1 = 3.5$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

3 (a) Given that $x = \tan(3y + 1)$:

(i) find $\dfrac{dx}{dy}$ in terms of y; *(2 marks)*

(ii) find the value of $\dfrac{dy}{dx}$ when $y = -\dfrac{1}{3}$. *(2 marks)*

(b) Sketch the graph of $y = \tan^{-1} x$. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

4 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = 3\cos\tfrac{1}{2}x, \qquad \text{for } 0 \leqslant x \leqslant 2\pi$$

$$\mathrm{g}(x) = |x|, \qquad \text{for all real values of } x$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 1$, giving your answer in an exact form. *(2 marks)*

(c) (i) Write down an expression for $\mathrm{gf}(x)$. *(1 mark)*

(ii) Sketch the graph of $y = \mathrm{gf}(x)$ for $0 \leqslant x \leqslant 2\pi$. *(3 marks)*

(d) Describe a sequence of two geometrical transformations that maps the graph of $y = \cos x$ onto the graph of $y = 3\cos\tfrac{1}{2}x$. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

5 (a) Find $\int \frac{1}{3+2x}\,\mathrm{d}x$. *(2 marks)*

(b) By using integration by parts, find $\int x \sin\frac{x}{2}\,\mathrm{d}x$. *(4 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

6 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^{0.4} \cos\sqrt{3x+1}\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

(b) Use the substitution $u = 3x + 1$ to find the exact value of $\int_0^1 x\sqrt{3x+1}\,\mathrm{d}x$. *(6 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

7 (a) Solve the equation $\sec x = -5$, giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. *(3 marks)*

(b) Show that the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

can be written in the form

$$\sec^2 x = 25$$

(4 marks)

(c) Hence, or otherwise, solve the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. *(3 marks)*

QUESTION PART REFERENCE

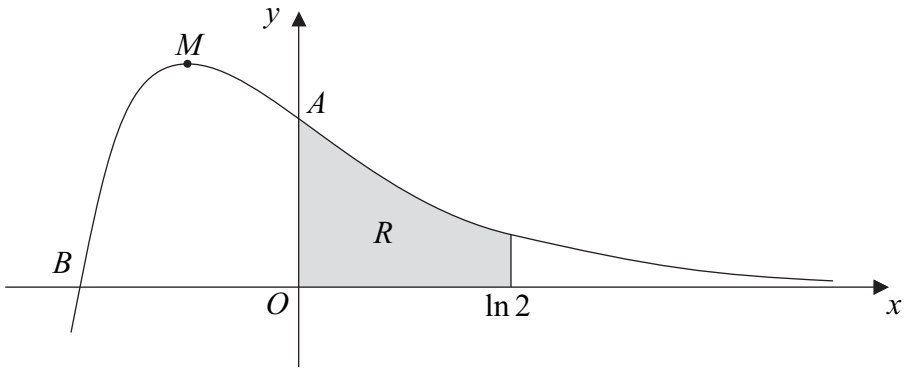
Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

8 (a) Given that $e^{-2x} = 4$, find the exact value of x. *(2 marks)*

(b) The diagram shows the curve $y = 4e^{-2x} - e^{-4x}$.



The curve crosses the y-axis at the point A, the x-axis at the point B, and has a stationary point at M.

(i) State the y-coordinate of A. *(1 mark)*

(ii) Find the x-coordinate of B, giving your answer in an exact form. *(3 marks)*

(iii) Find the x-coordinate of the stationary point, M, giving your answer in an exact form. *(3 marks)*

(iv) The shaded region R is bounded by the curve $y = 4e^{-2x} - e^{-4x}$, the lines $x = 0$ and $x = \ln 2$ and the x-axis.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in the form $\frac{p}{q}\pi$, where p and q are integers.

(7 marks)

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

END OF QUESTIONS

Copyright © 2011 AQA and its licensors. All rights reserved.