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General Certificate of Education (A-level)
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Mathematics **MPC4**

(Specification 6360)

Pure Core 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$\mathrm{f}\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^2-8\left(-\frac{1}{2}\right)-7$ $=-3$	M1 A1	2	Evaluate $\mathrm{f}\left(-\frac{1}{2}\right)$, not long division.
(b)(i)	$\mathrm{g}\left(-\frac{1}{2}\right)=0 \Rightarrow -3+d=0$ $d=3 \Rightarrow \mathrm{g}(x)=2x^3+2x^2-8x-7+3$ $\mathrm{g}(x)=2x^3+2x^2-8x-4$	B1	1	Or $\mathrm{f}\left(-\frac{1}{2}\right)+d=0$ All steps seen with conclusion **AG** Allow verification with $-\frac{1}{4}+\frac{1}{4}+4-4=0$ seen, and conclusion ; therefore factor
(ii)	$\mathrm{g}(x)=2x^3+x^2-8x-4=(2x+1)(x^2-4)$ $=(2x+1)(x+2)(x-2)$	B1	1	$a=-4$
(iii)	$2x^3-3x^2-2x=x(2x+1)(x-2)$ $\frac{(2x+1)(x+2)(x-2)}{x(2x+1)(x-2)}=\frac{x+2}{x}$ $\frac{\mathrm{g}(x)}{2x^3-3x^2-2x}=1+\frac{2}{x}$	M1 m1 A1	3	Clear attempt to factorise denominator; 3 factors needed. At least one correct factor cancelled CSO part (a)(iii) NMS is 0/3
	Total		**7**	
(b)(iii)	Alternative $\frac{\mathrm{g}(x)}{2x^3-3x^2-2x}=1+\frac{4x^2-6x-4}{2x^3-3x^2-2x}$ $=1+\frac{2(2x^2-3x-2)}{2x^3-3x^2-2x}$ $=1+\frac{2}{x}$	M1 A1 A1	**3**	$1+\frac{\text{quadratic}}{2x^3-3x^2-2x}$

Q	Solution	Marks	Total	Comments
2 (a)	$7x-1=A(1+3x)+B(3-x)$	M1		
	$x=3 \qquad x=-\frac{1}{3}$	m1		Use two values of x to find A **and** B. Or solve $A+3B=-1 \qquad 3A-B=7$ Or cover up rule
	$A=2 \qquad B=-1$	A1	3	
(b) (i)	$\frac{1}{1+3x}=(1+3x)^{-1}$			
	$=1+(-1)3x+\frac{1}{2}(-1)(-2)(3x)^2$	M1		Condone missing brackets
	$=1-3x+9x^2$	A1		
	$\frac{1}{3-x}=(3-x)^{-1}=\frac{1}{3}\left(1-\frac{x}{3}\right)^{-1}$	B1		
	$\left(1-\frac{x}{3}\right)^{-1}=1+(-1)\left(-\frac{x}{3}\right)+kx^2$	M1		Condone missing brackets
	$=1+\frac{x}{3}+\frac{x^2}{9}$	A1		
	$\frac{7x-1}{3+8x-3x^2}=$ $2\times\frac{1}{3}\times\left(1+\frac{x}{3}+\frac{x^2}{9}\right)-1\times\left(1-3x+9x^2\right)$	M1		Attempt to use PFs to combine expansions, or expand $(7x-1)(3-x)^{-1}(1+3x)^{-1}$
	$=-\frac{1}{3}+\frac{29}{9}x-\frac{241}{27}x^2$	A1	7	and simplify to $a+bx+cx^2$
(ii)	0.4 is outside the range of validity, because $0.4>\frac{1}{3}$.	B1	1	OE Accept $0.4>\frac{1}{3}$
	Total		**11**	

Q	Solution	Marks	Total	Comments
3 (a)(i)	$R = \sqrt{13}$	B1		Accept 3.6 or better
	$\tan\alpha = \frac{2}{3}$	M1		OE
	$\alpha = 33.7°$	A1	**3**	
(ii)	minimum value $= -\sqrt{13}$	B1ft		Accept -3.6 or better; ft R
	when $x - \alpha = \cos^{-1}(-1)$	M1		NMS 0/2
	$x = 213.7°$	A1	**3**	Calculus used 0/2
(b)(i)	LHS $= \frac{\cos x}{\sin x} - 2\sin x\cos x$	M1		Express $\cot x - \sin 2x$ in terms of $\sin x$ and $\cos x$; ACF
	$= \frac{\cos x}{\sin x}\left(1 - 2\sin^2 x\right)$	m1		Factor out $\frac{\cos x}{\sin x}$ and $1 - 2\sin^2 x$
	$= \cot x\cos 2x$	A1	**3**	All correct
(ii)	$\cot x - \sin 2x = 0$			
	$\cot x\cos 2x = 0$			
	$\cot x = 0$ or $\cos 2x = 0$	M1		Both equations correct
	$2x = 90°$ $(270°)$	m1		Condone missing 270°
	$x = 90°$, $45°$, $135°$	A1	**3**	All correct
	Total		**12**	
3 (b) (i)	Alternatives			
	RHS $= \cot x\cos 2x$			
	$= \frac{\cos x}{\sin x}\left(1 - 2\sin^2 x\right)$	M1		Express $\cot x\cos 2x$ in terms of $\cos x$ and $\sin x$, $\cos 2x$ ACF
	$= \frac{\cos x}{\sin x} - 2\sin x\cos x$	m1		$\cos 2x = 1 - 2\sin^2 x$ and multiply out and simplify.
	$= \cot x - \sin 2x$	A1	**3**	All correct.
	$\cot x\left(1 - \cos 2x\right) - \sin 2x = 0$			Rearrange to expression $= 0$ and factor out $\cot x$;
	$\frac{\cos x}{\sin x}\left(1 - \left(1 - 2\sin^2 x\right)\right) - 2\sin x\cos x = 0$	M1		Express $\cot x, \cos 2x$ and $\sin 2x$ in terms of $\sin x$ and $\cos x$, ACF
	$\frac{\cos x}{\sin x}\left(2\sin^2 x\right) - 2\sin x\cos x = 0$	m1		$\cos 2x = 1 - 2\sin^2 x$ used
	$2\sin x\cos x - 2\sin x\cos x = 0$	A1	**3**	Simplified, with all correct

3 (b)(ii)	Alternative $\cot x - \sin 2x = \dfrac{\cos x}{\sin x} - 2\sin x\cos x = 0$ $\cos x\left(\dfrac{1}{\sin x} - 2\sin x\right) = 0$ $\cos x = 0$ or $1 - 2\sin^2 x = 0$ $\sin x = (\pm)\dfrac{1}{\sqrt{2}}$ $x = 90°$, $45°$, $135°$	M1 m1 A1	**3**	Both equations

Q	Solution	Marks	Total	Comments
4 (a)(i)	$2x - 2y\frac{dy}{dx} = 0$	M1		Correct differentiation
	$\frac{dy}{dx} = \frac{x}{y}$			
	at (p, q) $\frac{dy}{dx} = \frac{p}{q}$	A1	**2**	(p, q) substituted into correct derivative or $x = p$ $y = q$ stated **AG**
(ii)	tangent at (p, q) $y - q = \frac{p}{q}(x - p)$	B1		ACF
	tangent at $(p, -q)$ $y - (-q) = \frac{-p}{q}(x - p)$	B1		ACF
	add $2y = 0$	M1		Solve tangent equations for y .
	conclusion $y = 0 \Rightarrow$ intersect on Ox	A1	**4**	Conclusion required
(b)	$x^2 = t^2 + 4 + \frac{4}{t^2}$ $y^2 = t^2 - 4 + \frac{4}{t^2}$	M1		Attempt to square *x* and *y* and subtract.
	$x^2 - y^2 = 8$	A1	**2**	All correct AG Allow $8 = 8$
	Total		**8**	

4(a)(i)	Alternative $y=\sqrt{x^2-8}\qquad \frac{dy}{dx}=\frac{1}{2}\times 2x\left(x^2-8\right)^{-\frac{1}{2}}=\frac{x}{y}$ $=\frac{p}{q}$	M1 A1	**2**	
(a)(i) **(ii)**	Alternative $\frac{dy}{dt}=1+\frac{2}{t^2}\qquad \frac{dx}{dt}=1-\frac{2}{t^2}$ $\frac{dy}{dx}=\frac{1+\frac{2}{t^2}}{1-\frac{2}{t^2}}=\frac{t+\frac{2}{t}}{t-\frac{2}{t}}=\frac{x}{y}$ at $(p,q)\quad \frac{dy}{dx}=\frac{p}{q}$ tangent at $(p,q)\qquad y-q=\frac{p(x-p)}{q}$ tangent at $(p,-q)\qquad y-(-q)=\frac{-p(x-p)}{q}$ When $y=0\quad \frac{-q^2}{p}=x-p$ and $\frac{q^2}{-p}=x-p$ $x=p-\frac{q^2}{p}$ is on both lines, so intersect on x axis	M1 A1 B1 B1 M1 A1	 **2** **4**	Attempt parametric derivatives and use chain rule. (p,q) substituted into correct derivative. ACF ACF Substitute $y=0$ into both candidate's tangents and solve for x. Conclusion
(b)	$x+y=2t\qquad x-y=\frac{4}{t}$ $(x-y)(x+y)=2t\times\frac{4}{t}$ $x^2-y^2=8$	M1 A1	**2**	Attempt to eliminate t

Q	Solution	Marks	Total	Comments
5(a)	$\int x\left(x^2+3\right)^{\frac{1}{2}}\ \mathrm{d}x = p\left(x^2+3\right)^{\frac{3}{2}}$	M1		By inspection or substitution
	$= \frac{1}{3}\left(x^2+3\right)^{\frac{3}{2}} \quad (+C)$	A1	**2**	
(b)	$\int \mathrm{e}^{2y}\ \mathrm{d}y = \int x\sqrt{x^2+3}\ \mathrm{d}x$	B1		Correct separation and notation Condone missing integral signs
	$\frac{1}{2}\mathrm{e}^{2y}$	B1		
	$= \frac{1}{3}\left(x^2+3\right)^{\frac{3}{2}} + C$	M1		Equate to result from (a) with constant.
	$\frac{1}{2} = \frac{1}{3}\times 4^{\frac{3}{2}} + C$	m1		Use $(1,0)$ to find constant.
	$C = -\frac{13}{6}$	A1		CAO
	$2y = \ln\left(\frac{2}{3}\left(x^2+3\right)^{\frac{3}{2}} - \frac{13}{3}\right)$	m1		Solve for y , taking logs correctly.
	$y = \frac{1}{2}\ln\left(\frac{2}{3}\left(x^2+3\right)^{\frac{3}{2}} - \frac{13}{3}\right)$	A1	**7**	CSO
	Total		**9**	

Q	Solution	Marks	Total	Comments
6 (a)(i)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{bmatrix} 8 \\ -4 \\ -6 \end{bmatrix} - \begin{bmatrix} 3 \\ 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix} = 5\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$	B1	**1**	Must see $\overrightarrow{OC} - \overrightarrow{OA}$ in correct components. $n = 5$
(ii)	$\overrightarrow{BC} = \begin{bmatrix} 3 \\ -2 \\ -6 \end{bmatrix}$	B1		$\overrightarrow{BC}$ or $\overrightarrow{CB}$ correct
	$5\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \bullet \begin{bmatrix} 3 \\ -2 \\ -6 \end{bmatrix} = 5\sqrt{2}\sqrt{49}\cos ACB$	M1		Correct form of formula using consistent vectors; condone use of θ or a wrong angle and a missing multiple of 5
	$5(3+2) = 5\sqrt{2}\sqrt{49}\cos ACB$	A1		Correct scalar product and moduli.
	$\cos ACB = \dfrac{5}{\sqrt{2}\times 7} = \dfrac{5\sqrt{2}}{2\times 7} = \dfrac{5\sqrt{2}}{14}$	A1	**4**	AG Must see, or rearrangement $\cos ACB = \dfrac{5}{\sqrt{2}\times 7}$ or $\dfrac{25}{35\sqrt{2}}$
(b)	vector equation $\mathbf{r} = \begin{bmatrix} 3 \\ 1 \\ -6 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix}$	M1		$\mathbf{a} + \lambda\mathbf{d}$
		A1	**2**	OE
(c)(i)	$\begin{bmatrix} 3 \\ 1 \\ -6 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ p \end{bmatrix}$	M1		Equate vector equations for *AC* and *BD*. OE
	$3 + 5\lambda = 5 + \mu$ $1 - 5\lambda = -2 + \mu$	M1		Set up equations and solve for μ; must find a value for μ
	$\mu = \dfrac{1}{2}$	A1		
	$-6 = \mu p \Rightarrow p = -12$	A1	**4**	
(ii)	$\overrightarrow{AB} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$ $\overrightarrow{CD} = \begin{bmatrix} -2 \\ 3 \\ -6 \end{bmatrix}$	M1		Clear attempt to find the vectors of the sides.
	$\overrightarrow{AD} = \begin{bmatrix} 3 \\ -2 \\ -6 \end{bmatrix}$ $\overrightarrow{BC} = \begin{bmatrix} 3 \\ -2 \\ -6 \end{bmatrix}$	A1		All vectors correct
		m1		Find the lengths of the sides, or state they all $= \sqrt{49}$ if all correct.
	All sides are of same length, 7; hence rhombus.	A1	**4**	Each side $= 7$ and conclusion. Or adjacdnt sides $= 7$ and opposite sides are parallel.
	Total		**15**	

(c)(ii)	Alternative $\overrightarrow{AC} \bullet \overrightarrow{BD} = 5 - 5$ $= 0 \Rightarrow$ $\overrightarrow{AC}$ and $\overrightarrow{BD}$ are perpendicular $\mu = \frac{1}{2} \Rightarrow \lambda = \frac{1}{2} \Rightarrow$ intersection is at midpoint of AC and BD Diagonals bisect each other at right angles; hence rhombus, with all sides equal to 7	M1 A1 M1 A1		Calculate scalar product of $\overrightarrow{AC}$ and $\overrightarrow{BD}$ $= 0$ from correct $\overrightarrow{AC}$ and $\overrightarrow{BD}$ and conclusion Find value of λ and attempt to use in argument about point of intersection Fully correct conclusion. Must show diagonals bisect

Q	Solution	Marks	Total	Comments
7				
(a)(i)	$t = 0 \quad N = 50$	B1	1	
(ii)	$t = 24 \quad N = 345$	B1	1	Must be 345 (not 345.2534..)
(iii)	$1 + 9e^{-\frac{t}{8}} = \frac{500}{400} \Rightarrow 9e^{-\frac{t}{8}} = \frac{1}{4}$	M1		Correct algebra seen
	$e^{\frac{t}{8}} = 36$	m1		Or $e^{-\frac{t}{8}} = \frac{1}{36}$
	$t = 8\ln 36$	A1	3	or $t = 16\ln 6$
(b)				
(i)	$\frac{dN}{dt} = -500\left(1 + 9e^{-\frac{t}{8}}\right)^{-2}\left(-\frac{9}{8}e^{-\frac{t}{8}}\right)$	M1 A1		Clear attempt at chain rule or quotient rule.
	$= -500\left(-\frac{1}{8}\left(\frac{500}{N} - 1\right)\right)\left(\frac{500}{N}\right)^{-2}$	m1		Use $e^{-\frac{t}{8}} = \frac{1}{9}\left(\frac{500}{N} - 1\right)$ to
	$= \frac{N^2}{500}\left(\frac{1}{8}\left(\frac{500}{N} - 1\right)\right)$			eliminate $e^{-\frac{t}{8}}$.
	$\frac{dN}{dt} = \frac{N}{4000}(500 - N)$	A1	4	Correct algebra to AG
(ii)	$\frac{d}{dN}\left(500N - N^2\right) = 500 - 2N$	M1		Differentiate and attempt to find N at max value
	$500 - 2N = 0 \Rightarrow N = 250$	A1		Condone $\frac{d^2}{dt^2}$ for $\frac{d}{dN}$
	$9e^{-\frac{T}{8}} = 1$	m1		
	$e^{\frac{T}{8}} = 9$			
	$T = 8\ln 9 = 17(.577)$	A1	4	$T = 17$ or better CSO Accept 17, 18, 17.5, 17.6
	Total		**13**	
	TOTAL		**75**	
(b)(ii)	Alternative, by inspection Max of $N(500 - N)$ occurs at $N = 250$	B2		

(b)(i)	Alternatives Alternative 1 implicit differentiation $e^{-\frac{t}{8}} = \frac{500-N}{9N}$ $\frac{dt}{dN}\left(-\frac{1}{8}e^{-\frac{t}{8}}\right) = -\frac{500}{9N^2}$ use $e^{-\frac{t}{8}} = \frac{1}{9}\left(\frac{500}{N}-1\right)$ to get $\frac{dt}{dN} = \frac{4000}{9N^2} \times \frac{9N}{500-N}$ $\frac{dN}{dt} = \frac{N}{4000}(500-N)$	M1 A1 m1 A1	 **4**	Correct expressions for $e^{-\frac{t}{8}}$ and attempt to use implicit differentiation Fully correct Attempt to eliminate $e^{-\frac{t}{8}}$ using correct expression
	Alternative 2 explicit differentiation $t = -8\ln\left(\frac{500-N}{9N}\right)$ $\frac{dt}{dN} = -8\left(\frac{(500-N)\left(\frac{-1}{9N^2}\right)-\frac{1}{9N}}{\left(\frac{500-N}{9N}\right)}\right)$ $= \frac{8}{9N}\left(9+\frac{9N}{500-N}\right)$ $= \frac{8}{9N}\left(\frac{4500}{500-N}\right)$ $\frac{dN}{dt} = \frac{N}{4000}(500-N)$	M1 A1 m1 A1	 **4**	Correct expression for t and attempt at differentiation with use of chain rule and product for ln derivative. Clear fractions within fractions All correct
	Or $t = -8(\ln(500-N) - \ln(9N))$ $\frac{dt}{dN} = -8\left(\frac{-1}{500-N} - \frac{9}{9N}\right)$ $= 8\left(\frac{1}{500-N} + \frac{1}{N}\right)$ $= 8\left(\frac{N+500-N}{N(500-N)}\right)$ $= \frac{4000}{N(500-N)} \Rightarrow \frac{dN}{dt} = \frac{4000}{N(500-N)}$	M1 A1 m1 A1	 **4**	Correct expression for t and ln derivatives, condone sign errors Common denominator to combine fractions All correct
	Alternative 3 solve differential equation			

	$\int \frac{\mathrm{d}N}{N(500-N)} = \int \frac{\mathrm{d}t}{4000}$	M1		Separate variables, and attempt to form partial fractions and integrate to ln terms $=kt+C$
	$\int \frac{1}{500}\left(\frac{1}{N}+\frac{1}{500-N}\right)\mathrm{d}N = \int \frac{\mathrm{d}t}{4000}$ $\ln N - \ln(500-N) = \frac{500}{4000}t + C$	A1		
	$(t=0 \quad N=50) \qquad C = \ln\left(\frac{1}{9}\right)$ $\ln\left(\frac{9N}{500-N}\right) = \frac{1}{8}t \Rightarrow \frac{9N}{500-N} = e^{\frac{1}{8}t}$	m1		Use $(50,0)$ to find C and obtain $\mathrm{e}^{\frac{1}{8}t} = \mathrm{f}(N)$
	$N\left(9+e^{\frac{1}{8}t}\right) = 500e^{\frac{1}{8}t}$ $N = \frac{500e^{\frac{1}{8}t}}{9+e^{\frac{1}{8}t}} = \frac{500}{1+9e^{-\frac{1}{8}t}}$	A1		Manipulate correctly to original given equation.