



Oxford Cambridge and RSA

Tuesday 7 June 2022 – Afternoon

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi \right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^n C_r p^r q^{n-r}$ where $q = 1 - p$

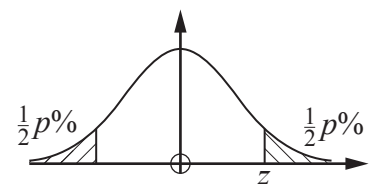
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

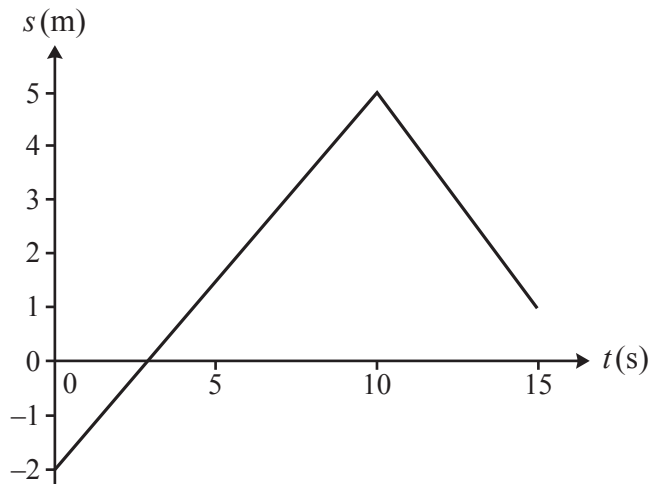
$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

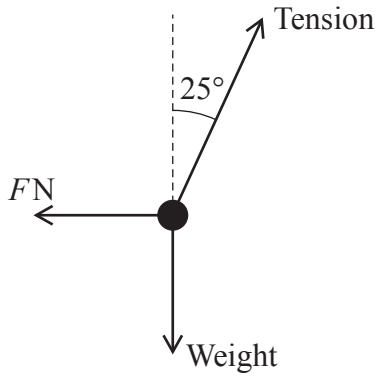
Section A (24 marks)

- 1 A particle moves along a straight line. The displacement s m at time t s is shown in the displacement-time graph below. The graph consists of straight line segments joining the points $(0, -2)$, $(10, 5)$ and $(15, 1)$.



- (a) Find the distance travelled by the particle in the first 15 s. [2]
- (b) Calculate the velocity of the particle between $t = 10$ and $t = 15$. [2]
- 2 Express $\frac{13-x}{(x-3)(x+2)}$ in partial fractions. [3]
- 3 (a) Sketch the graph of $y = \arctan x$ where x is in radians. [2]
- (b) **In this question you must show detailed reasoning.**
- Find all points of intersection of the curves $y = 3 \sin x \cos x$ and $y = \cos^2 x$ for $-\pi \leq x \leq \pi$. [6]
- 4 Using an appropriate expansion show that, for sufficiently small values of x ,
- $$\frac{1-x}{(2+x)^2} \approx \frac{1}{4} - \frac{1}{2}x + \frac{7}{16}x^2. \quad [4]$$

- 5 A sphere of mass 3 kg hangs on a string. A horizontal force of magnitude F N acts on the sphere so that it hangs in equilibrium with the string making an angle of 25° to the vertical. The force diagram for the sphere is shown below.

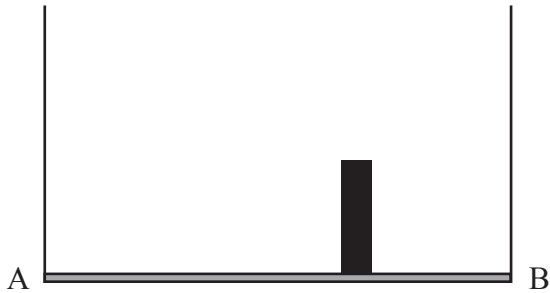


- (a) Sketch the triangle of forces for these forces. [2]
- (b) Hence or otherwise determine each of the following:
- the tension in the string
 - the value of F . [3]

Answer **all** the questions.

Section B (76 marks)

- 6 A shelf consists of a horizontal uniform plank AB of length 0.8m and mass 5 kg with light inextensible vertical strings attached at each end. A stack of bricks each of mass 2.3 kg is placed on the plank as shown in the diagram.



- (a) Explain the meaning of each of the following modelling assumptions.
- The stack of bricks is modelled as a particle.
 - The plank is modelled as uniform.
- [2]

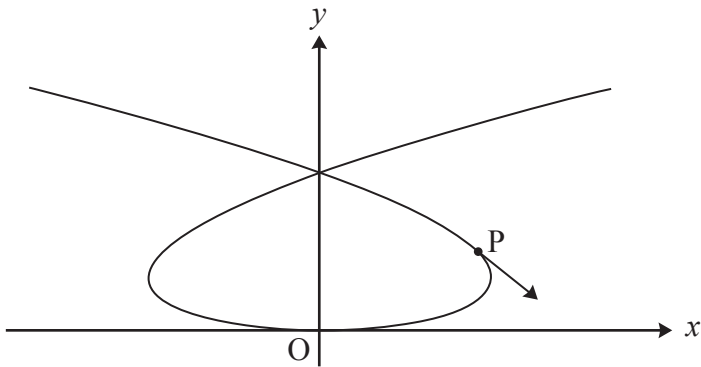
Either of the strings will break if the tension exceeds 75 N.

- (b) Find the greatest number of bricks that can be placed at the centre of the plank without breaking the strings. [2]
- (c) Find an expression for the moment about A of the weight of a stack of n bricks when the stack is at a distance of x m from A. State the units for your answer. [2]
- (d) Calculate the greatest distance from A that the largest stack of bricks can be placed without a string breaking. [3]
- 7 In this question the x - and y -directions are horizontal and vertically upwards respectively and the origin is on horizontal ground.

A ball is thrown from a point 5 m above the origin with an initial velocity $\begin{pmatrix} 14 \\ 7 \end{pmatrix} \text{ m s}^{-1}$.

- (a) Find the position vector of the ball at time t s after it is thrown. [3]
- (b) Find the distance between the origin and the point at which the ball lands on the ground. [3]

- 8 A particle moves in the x - y plane so that its position at time t s is given by $x = t^3 - 8t$, $y = t^2$ for $-3.5 < t < 3.5$. The units of distance are metres. The graph shows the path of the particle and the direction of travel at the point P (8, 4).



- (a) Find $\frac{dy}{dx}$ in terms of t . [3]
- (b) Hence show that the value of $\frac{dy}{dx}$ at P is -1 . [2]
- (c) Find the time at which the particle is travelling in the direction opposite to that at P. [2]
- (d) Find the cartesian equation of the path, giving x^2 as a function of y . [3]
- 9 In this question, the vectors \mathbf{i} and \mathbf{j} are directed east and north respectively.

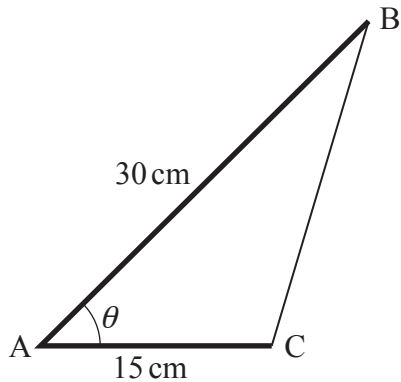
The velocity \mathbf{v} m s^{-1} of a particle at time t s is given by $\mathbf{v} = kt^2\mathbf{i} + 6t\mathbf{j}$, where k is a positive constant. The magnitude of the acceleration when $t = 2$ is 10 m s^{-2} .

- (a) Calculate the value of k . [4]

The particle is at the origin when $t = 0$.

- (b) Determine an expression for the position vector of the particle at time t . [2]
- (c) Determine the time when the particle is directly north-east of the origin. [2]

- 10 A triangle ABC is made from two thin rods hinged together at A and a piece of elastic which joins B and C. AB is a 30 cm rod and AC is a 15 cm rod. The angle BAC is θ radians as shown in the diagram.



The angle θ increases at a rate of 0.1 radians per second.

Determine the rate of change of the length BC when $\theta = \frac{1}{3}\pi$. [8]

- 11 Given that k is a positive constant, show that $\int_k^{2k} \frac{2}{(2x+k)^2} dx$ is inversely proportional to k . [6]

- 12 Prove by contradiction that 3 is the only prime number which is 1 less than a square number. [4]

- 13** A toy train consists of an engine of mass 0.5 kg pulling a coach of mass 0.4 kg. The coupling between the engine and the coach is light and inextensible. The train is pulled along with a string attached to the front of the engine.

At first, the train is pulled from rest along a horizontal carpet where there is a resistance to motion of 0.8 N on each part of the train. The string is horizontal, and the tension in the string is 5 N.

- (a) Determine the velocity of the train after 1.5 s. [4]

The train is then pulled up a track inclined at 20° to the horizontal. The string is parallel to the track and the tension in the string is P N. The resistance on each part of the train along the track is R N.

- (b) Draw a diagram showing all the forces acting on the train modelled as two connected particles. [3]

- (c) Find the equation of motion for the train modelled as a single particle. [2]

- (d) The acceleration of the train when $P = 5.5$ is double the acceleration when $P = 5$.

Calculate the value of R . [3]

- 14 Alex places a hot object into iced water and records the temperature $\theta^\circ\text{C}$ of the object every minute. The temperature of an object t minutes after being placed in iced water is modelled by $\theta = \theta_0 e^{-kt}$ where θ_0 and k are constants whose values depend on the characteristics of the object.

The temperature of Alex's object is 82°C when it is placed into the water. After 5 minutes the temperature is 27°C .

- (a) Find the values of θ_0 and k that best model the data. [3]
- (b) Explain why the model may **not** be suitable in the long term if Alex does not top up the ice in the water. [1]
- (c) Show that the model with the values found in part (a) can be written as $\ln\theta = a - bt$ where a and b are constants to be determined. [2]

Ben places a different object into iced water at the same time as Alex. The model for Ben's object is $\ln\theta = 3.4 - 0.08t$.

- (d) Determine each of the following:
- the initial temperature of Ben's object
 - the rate at which Ben's object is cooling initially. [4]
- (e) According to the models, there is a time at which both objects have the same temperature. Find this time and the corresponding temperature. [3]

END OF QUESTION PAPER

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