OCR
Oxford Cambridge and RSA

Thursday 19 May 2022 – Afternoon

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

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You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 Find the term in x^3 in the binomial expansion of $(3-2x)^5$. **[2]**

2 **In this question you must show detailed reasoning.**

The cubic polynomial f(x) is defined by $\mathrm{f}(x) = 5x^3 - 4x^2 + ax - 2$, where a is a constant.

You are given that $(x-2)$ is a factor of f(x).

(a) Find the value of a. **[2]**

(b) Find all the factors of f(x). **[3]**

3 The diagram in the Printed Answer Booklet shows part of the graph of $y = x^2 - 4x + 3$.

(a) It is required to solve the equation $x^2 - 3x + 1 = 0$ graphically by drawing a straight line with equation $y = mx + c$ on the diagram, where m and c are constants.

Find the values of m and c. **[2]**

(b) Use the graph to find approximate values of the roots of the equation $x^2 - 3x + 1 = 0$. **[2]**

(c) By shading, or otherwise, indicate clearly the regions where **all** of the following inequalities are satisfied. You should use the values of m and c found in part **(a)**.

$x \geqslant 0$ $\quad$ $x \leqslant 4$ $\quad$ $y \leqslant x^2 - 4x + 3$ $\quad$ $y \geqslant mx + c$ **[3]**

4 **In this question you must show detailed reasoning.**

Solve the following equations, for $0° \leqslant x \leqslant 360°$.

(a) $2\tan x + 1 = 4$ **[3]**

(b) $5\sin x - 1 = 2\cos^2 x$ **[5]**

5 The gradient of a curve is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2 - 3x$. The curve passes through the point (6, 20).

(a) Determine the equation of the curve. **[4]**

(b) Hence determine $\int_1^p y\,\mathrm{d}x$ in terms of the constant p. **[4]**

6 During some research the size, P, of a population of insects, at time t months after the start of the research, is modelled by the following formula.

$P = 100\,\mathrm{e}^{t}$

(a) Use this model to answer the following.

(i) Find the value of P when $t = 4$. **[1]**

(ii) Find the value of t when the population is 9000. **[2]**

(b) It is suspected that a more appropriate model would be the following formula.

$P = ka^{t}$ where k and a are constants.

(i) Show that, using this model, the graph of $\log_{10}P$ against t would be a straight line. **[2]**

Some observations of t and P gave the following results.

t	1	2	3	4	5
P	100	500	1800	7000	19000
$\log_{10}P$	2.00	2.70	3.26	3.85	4.28

(ii) On the grid in the Printed Answer Booklet, draw a line of best fit for the data points $(t, \log_{10}P)$ given in the table. **[2]**

(iii) Hence estimate the values of k and a. **[4]**

7 **(a)** **In this question you must show detailed reasoning.**

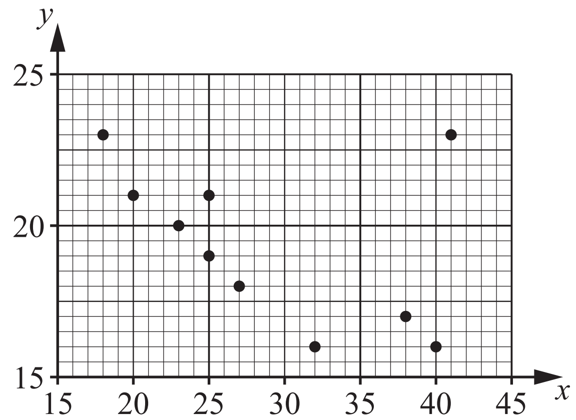
Find the range of values of the constant m for which the simultaneous equations $y = mx$ and $x^2 + y^2 - 6x - 2y + 5 = 0$ have real solutions. **[5]**

(b) Give a geometrical interpretation of the solution in the case where $m = 2$. **[3]**

Section B: Statistics

Answer **all** the questions.

8 A random sample of 10 students from a college was chosen. They were asked how much time, x hours, they spent studying, and how much money, £y, they earned, in a typical week during term time. The results are shown in the scatter diagram.



(a) Comment on the relationship shown by the diagram between hours spent studying and money earned, during term time, by these 10 students. **[2]**

The coordinates of the points in the diagram are (18, 23), (20, 21), (23,20), (25, 19), (25, 21), (27,18), (32, 16), (38, 17), (40, 16) and (41, 23).

(b) Find the mean and standard deviation of the number of hours spent per week studying during term time by these 10 students. **[2]**

9 Last year, market research showed that 8% of adults living in a certain town used a particular local coffee shop. Following an advertising campaign, it was expected that this proportion would increase. In order to test whether this had happened, a random sample of 150 adults in the town was chosen.

The random variable X denotes the number of these 150 adults who said that they used the local coffee shop.

(a) **(i)** Assuming that the proportion of adults using the local coffee shop is unchanged from the previous year, state a suitable binomial distribution with which to model the variable X. **[1]**

(ii) The probabilities given by this model are the terms of the binomial expansion of an expression of the form $(a+b)^n$.

Write down this expression, using appropriate values of a, b and n. **[1]**

It was found that 18 of these 150 adults said that they use the local coffee shop.

(b) Test, at the 5% significance level, whether the proportion of adults in the town who use the local coffee shop has increased. **[7]**

It was later discovered by a statistician that the random sample of 150 adults had been chosen from shoppers in the town on a Friday and a Saturday.

(c) Explain why this suggests that the assumptions made when using a binomial model for X may not be valid in this context. **[1]**

10 The table shows the increases, between 2001 and 2011, in the percentages of employees travelling to work by various methods, in the Local Authorities (LAs) in the North East region of the UK.

Geography code	Local authority	Work mainly at or from home	Underground, metro, light rail or tram	Bus, minibus or coach	Driving a car or van	Passenger in a car or van	On foot
E06000047	**County Durham**	0.74%	0.05%	−1.50%	4.58%	−2.99%	−0.97%
E06000005	**Darlington**	0.26%	−0.01%	−3.25%	3.06%	−1.28%	0.99%
E08000020	**Gateshead**	−0.01%	−0.01%	−2.28%	4.62%	−2.35%	−0.18%
E06000001	**Hartlepool**	0.03%	−0.04%	−1.62%	4.80%	−2.38%	−0.26%
E06000002	**Middlesbrough**	−0.34%	−0.01%	−2.32%	2.19%	−1.33%	0.67%
E08000021	**Newcastle upon Tyne**	0.10%	−0.23%	−0.67%	−0.48%	−1.51%	1.75%
E08000022	**North Tyneside**	0.05%	0.54%	−1.18%	3.30%	−2.21%	−0.60%
E06000048	**Northumberland**	1.39%	−0.08%	−0.95%	3.50%	−2.37%	−1.44%
E06000003	**Redcar and Cleveland**	−0.02%	−0.01%	−2.09%	4.20%	−2.06%	−0.49%
E08000023	**South Tyneside**	−0.36%	2.03%	−3.05%	4.50%	−2.41%	−0.51%
E06000004	**Stockton-on-Tees**	0.14%	0.03%	−2.02%	3.52%	−2.01%	−0.15%
E08000024	**Sunderland**	0.17%	1.48%	−3.11%	4.89%	−2.21%	−0.52%

Increase in percentage of employees travelling to work by various methods

The first two digits of the Geography code give the type of each of the LAs:

06: Unitary authority
07: Non-metropolitan district
08: Metropolitan borough

(a) In what type of LA are the largest increases in percentages of people travelling by underground, metro, light rail or tram? **[1]**

(b) Identify two main changes in the pattern of travel to work in the North East region between 2001 and 2011. **[2]**

Now assume the following.

- The data refer to residents in the given LAs who are in the age range 20 to 65 at the time of each census.
- The number of people in the age range 20 to 65 who move into or out of each given LA, or who die, between 2001 and 2011 is negligible.

(c) Estimate the percentage of the people in the age range 20 to 65 in 2011 whose data appears in both 2001 and 2011. **[2]**

(d) In the light of your answer to part **(c)**, suggest a reason for the changes in the pattern of travel to work in the North East region between 2001 and 2011. **[1]**

11 Alex models the number of goals that a local team will score in any match as follows.

Number of goals	0	1	2	3	4	More than 4
Probability	$\frac{3}{25}$	$\frac{1}{5}$	$\frac{8}{25}$	$\frac{7}{25}$	$\frac{2}{25}$	0

The number of goals scored in any match is independent of the number of goals scored in any other match.

(a) Alex chooses 3 matches at random. Use the model to determine the probability of each of the following.

(i) The team will score a total of exactly 1 goal in the 3 matches. **[2]**

(ii) The numbers of goals scored in the first 2 of the 3 matches will be equal, but the number of goals scored in the 3rd match will be different. **[3]**

During the first 10 matches this season, the team scores a total of 31 goals.

(b) Without carrying out a formal test, explain briefly whether this casts doubt on the validity of Alex's model. **[1]**

END OF QUESTION PAPER