Oxford Cambridge and RSA

Wednesday 8 June 2022 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1} b + {}^n\mathrm{C}_2 a^{n-2} b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r} b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \tfrac{1}{2}at^2$$

Section A: Pure Mathematics

Answer **all** the questions.

1 Write the solution of the inequality $(x-2)(x+3) > 0$ using set notation. **[2]**

2 **In this question you must show detailed reasoning.**

Solve the equation $3x+1 = 4\sqrt{x}$. **[4]**

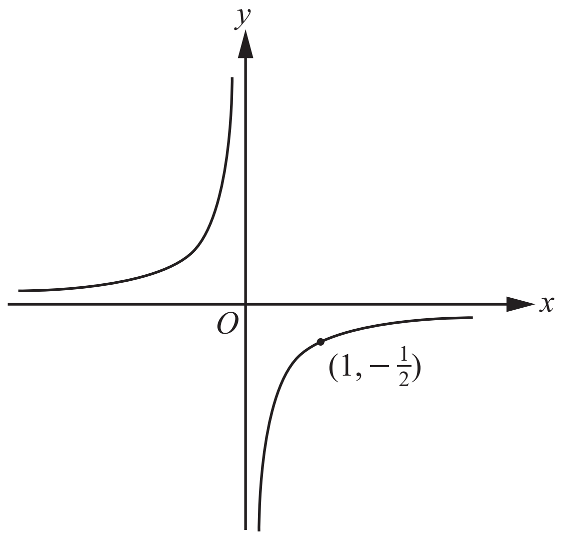
3 Give a counter example to disprove the following statement.

If x and y are both irrational then $x+y$ is irrational. **[2]**

4 The circle $x^2+y^2-6x+4y+k=0$ has radius 5.

Determine the value of k. **[3]**

5



The diagram shows a curve C for which y is inversely proportional to x. The curve passes through the point $\left(1, -\frac{1}{2}\right)$.

(a) **(i)** Determine the equation of the **gradient function** for the curve C. **[3]**

(ii) Sketch this **gradient function** on the axes in the Printed Answer Booklet. **[1]**

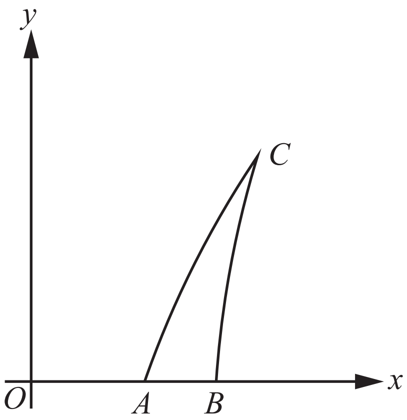
(b) The diagram indicates that the curve C has no stationary points.

State what feature of your sketch in part **(a)(ii)** corresponds to this. **[1]**

(c) The curve C is translated by the vector $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$.

Find the equation of the curve after it has been translated. **[2]**

6



The shape ABC shown in the diagram is a student's design for the sail of a small boat.

The curve AC has equation $y = 2\log_2 x$ and the curve BC has equation $y = \log_2\left(x - \frac{3}{2}\right) + 3$.

(a) State the x-coordinate of point A. **[1]**

(b) Determine the x-coordinate of point B. **[3]**

(c) By solving an equation involving logarithms, show that the x-coordinate of point C is 2. **[4]**

It is given that, correct to 3 significant figures, the area of the sail is 0.656 units^2.

(d) Calculate by how much the area is over-estimated or under-estimated when the curved edges of the sail are modelled as straight lines. **[4]**

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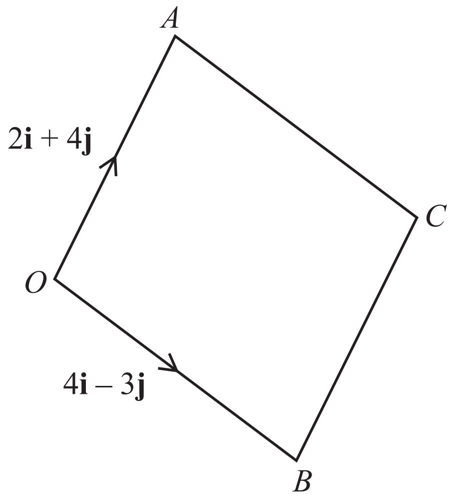


The diagram shows the parallelogram *OACB* where $\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j}$ and $\overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j}$.

(a) Show that $\cos AOB = -\dfrac{2\sqrt{5}}{25}$. **[5]**

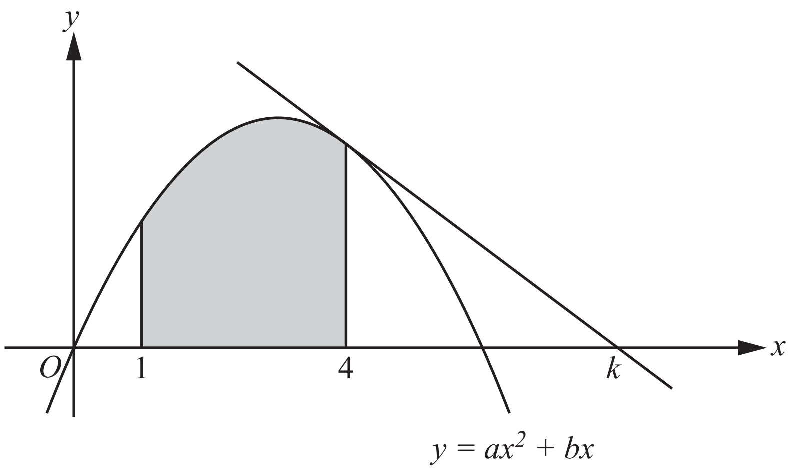
(b) Hence find the exact value of $\sin AOB$. **[2]**

(c) Determine the area of *OACB*. **[2]**

8 **(a)** The quadratic polynomial $ax^2 + bx$, where a and b are constants, is denoted by $\mathrm{f}(x)$.

Use differentiation from first principles to determine, in terms of a, b and x, an expression for $\mathrm{f}'(x)$. **[4]**

(b)



The diagram shows the quadratic curve $y = ax^2 + bx$, where a and b are constants. The shaded region is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

The tangent to the curve at $x = 4$ intersects the x-axis at the point with coordinates $(k, 0)$.

Given that the area of the shaded region is 9 units^2, and the gradient of this tangent is $-\frac{3}{4}$, determine the value of k. **[7]**

Section B: Mechanics

Answer **all** the questions.

9 Two forces $(3\mathbf{i}+2\mathbf{j})$ N and $\mathbf{F}$ N act on a particle P of mass 4 kg.

Given that the acceleration of P is $(-2\mathbf{i}+3\mathbf{j})\,\mathrm{m\,s^{-2}}$, calculate $\mathbf{F}$. **[2]**

10 A small ball B is projected vertically upwards from a point 2 m above horizontal ground. B is projected with initial speed $3.5\,\mathrm{m\,s^{-1}}$, and takes t seconds to reach the ground.

Find the value of t. **[3]**

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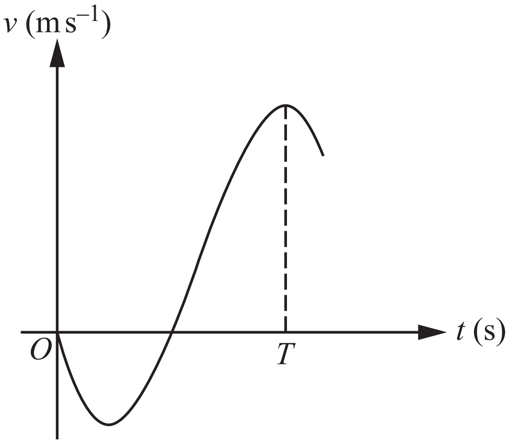

A particle P moves along the x-axis. At time t seconds, where $t \geqslant 0$, the velocity of P in the positive x-direction is $v\,\mathrm{m\,s^{-1}}$. It is given that $v = t(t-3)(8-t)$.

P attains its maximum velocity at time T seconds. The diagram shows part of the velocity-time graph for the motion of P.

(a) State the acceleration of P at time T. **[1]**

(b) **In this question you must show detailed reasoning.**

Determine the value of T. **[5]**

(c) Find the total distance that P travels between times $t = 0$ and $t = T$. **[3]**

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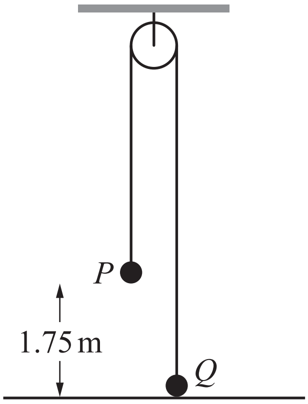

Particles P and Q, of masses 4 kg and 6 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is in equilibrium with P hanging 1.75 m above a horizontal plane and Q resting on the plane. Both parts of the string below the pulley are vertical (see diagram).

(a) Find the magnitude of the normal reaction force acting on Q. **[1]**

The mass of P is doubled, and the system is released from rest. You may assume that in the subsequent motion Q does not reach the pulley.

(b) Determine the magnitude of the force exerted on the pulley by the string before P strikes the plane. **[5]**

(c) Determine the total distance travelled by Q between the instant when the system is released and the instant when Q first comes momentarily to rest. **[4]**

When this motion is observed in practice, it is found that the total distance travelled by Q between the instant when the system is released and the instant when Q first comes momentarily to rest is less than the answer calculated in part **(c)**.

(d) State **one** factor that could account for this difference. **[1]**

END OF QUESTION PAPER

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