Oxford Cambridge and RSA

# Thursday 19 May 2022 – Afternoon

## AS Level Mathematics B (MEI)

### H630/01 Pure Mathematics and Mechanics

**Time allowed: 1 hour 30 minutes**

**You must have:**
- the Printed Answer Booklet
- a scientific or graphical calculator

*9119902547*

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

**ADVICE**
- Read each question carefully before you start your answer.

**Formulae AS Level Mathematics B (MEI) (H630)**

**Binomial series**

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

**Differentiation from first principles**

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

**Sample variance**

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

**The binomial distribution**

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of $X$ is $np$

**Kinematics**

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

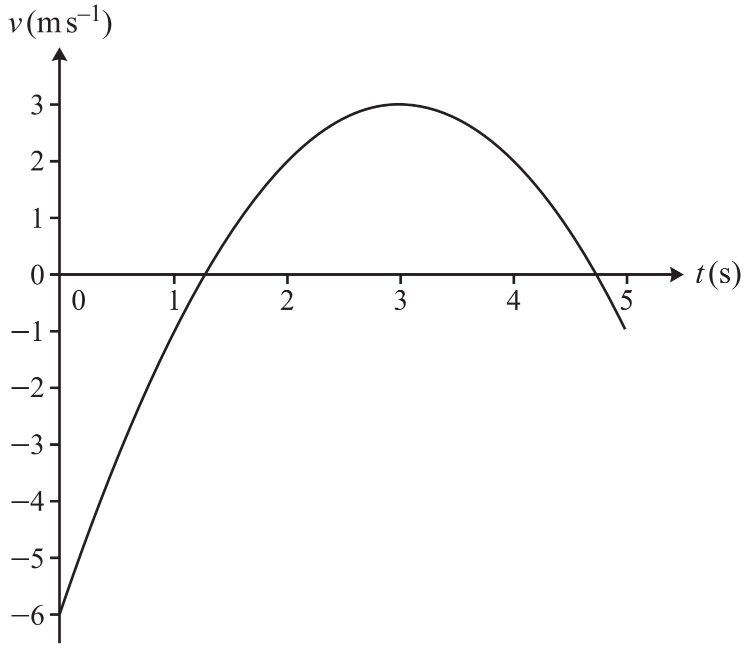
Answer **all** the questions.

**1** Rationalise the denominator of the fraction $\dfrac{2+\sqrt{n}}{3+\sqrt{n}}$, where $n$ is a positive integer. **[3]**

**2** **(a)** Determine the value of $\dfrac{100!}{98!}$. **[2]**

**(b)** Find the coefficient of $x^{98}$ in the expansion of $(1+x)^{100}$. **[1]**

**3** The velocity-time graph for the motion of a particle is shown below. The velocity $v\,\mathrm{m\,s^{-1}}$ at time $t\,\mathrm{s}$ is given by $v = -t^2 + 6t - 6$ where $0 \leqslant t \leqslant 5$.




**(a)** Find the times at which the velocity is $2\,\mathrm{m\,s^{-1}}$. **[2]**

**(b)** Write down the greatest speed of the particle. **[1]**

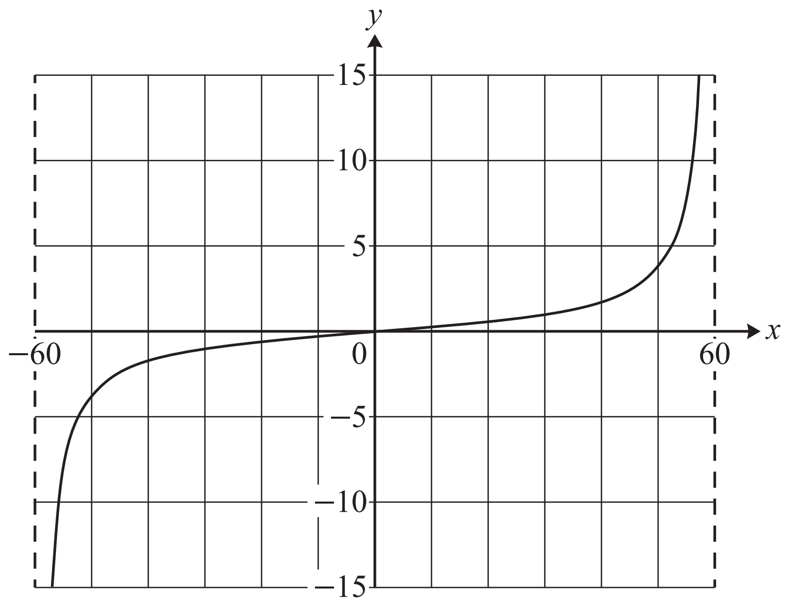
**4** The quadratic function $f(x)$ is given by $f(x) = x^2 - 3x + 2$.

**(a)** Write $f(x)$ in the form $(x+a)^2 + b$, where $a$ and $b$ are constants. **[2]**

**(b)** Write down the coordinates of the minimum point on the graph of $y = f(x)$. **[2]**

**(c)** Describe fully the transformation that maps the graph of $y = f(x)$ onto the graph of $y = (x+1)^2 - \frac{1}{4}$. **[2]**

**5** Part of the graph of $y = f(x)$ is shown below. The graph is the image of $y = \tan x°$ after a stretch in the $x$-direction.




**(a)** Find the equation of the graph. **[2]**

**(b)** Write down the period of the function $f(x)$. **[1]**

**(c)** **In this question you must show detailed reasoning.**

Find all the roots of the equation $f(x) = 1$ for $0° \leqslant x° \leqslant 360°$. **[3]**

**6** The gradient of a curve is given by the equation $\frac{dy}{dx} = 6x^2 - 20x + 6$. The curve passes through the point (2, 6).

**(a)** Find the equation of the curve. **[3]**

**(b)** Verify that the equation of the curve can be written as $y = 2(x+1)(x-3)^2$. **[2]**

**(c)** Sketch the curve, indicating the points where the curve meets the axes. **[3]**

**7** In this question the unit vectors **i** and **j** are directed east and north respectively.

A canal narrowboat of mass 9 tonnes is pulled by two ropes. The tensions in the ropes are $(450\mathbf{i} + 20\mathbf{j})$ N and $(420\mathbf{i} - 20\mathbf{j})$ N. The boat experiences a resistance to motion **R** of magnitude 300 N.

**(a)** Explain what it means to model the boat as a particle. **[1]**

The boat is travelling in a straight line due east.

**(b)** Find the equation of motion of the boat. **[2]**

**(c)** Find the acceleration of the boat giving your answer as a vector. **[1]**

**8** A team of volunteers donates cakes for sale at a charity stall. The number of cakes that can be sold depends on the price. A model for this is $y = 190 - 70x$, where $y$ cakes can be sold when the price of a cake is £$x$.

**(a)** Find how many cakes could be given away for free according to this model. **[1]**

The number of volunteers who are willing to donate cakes goes up as the price goes up. If the cakes sell for £1.20 they will donate 50 cakes, but if they sell for £2.40 they will donate 140 cakes. They use the linear model $y = mx + c$ to relate the number of cakes donated, $y$, to the price of a cake, £$x$.

**(b)** Find the values of the constants $m$ and $c$ for which this linear model fits the two data points. **[2]**

**(c)** Explain why the model is **not** suitable for very low prices. **[1]**

**(d)** The team would like to sell all the cakes that they donate.

Find the set of possible prices that the cakes could have to achieve this. **[3]**

**9** A tractor of mass 1800 kg uses a towbar to pull a trailer of mass 1000 kg on a level field. The tractor and trailer experience resistances to motion of 1600 N and 800 N respectively. The tractor provides a driving force of 6600 N.

**(a)** Draw a force diagram showing all the horizontal forces acting on the tractor and trailer. **[2]**

**(b)** Find the tension in the towbar. **[4]**

**10** A triangle has vertices A $(1, 4)$, B $(7, 0)$ and C $(-4, -1)$.

**(a)** Show that the equation of the line AC is $y = x + 3$. **[2]**

M is the midpoint of AB. The line AC intersects the $x$-axis at D.

**(b)** Determine the angle DMA. **[7]**

**11** A sports car accelerates along a straight road from rest. After 5 s its velocity is $9\,\mathrm{m\,s^{-1}}$.

In model A, the acceleration is assumed to be constant.

**(a)** Calculate the distance travelled by the car in the first 5 seconds according to model A. **[2]**

In model B, the velocity $v$ in $\mathrm{m\,s^{-1}}$ is given by $v = 0.05t^3 + kt$, where $t$ is the time in seconds after the start and $k$ is a constant.

**(b)** Find the value of $k$ which gives the correct value of $v$ when $t = 5$. **[2]**

**(c)** Using this value of $k$ in model B, calculate the acceleration of the car when $t = 5$. **[2]**

The car travels 16 m in the first 5 seconds.

**(d)** Show that model B, with the value of $k$ found in part **(b)**, better fits this information than model A does. **[3]**

**12** Below is a faulty argument that appears to show that the gradient of the curve $y = x^2$ at the point (3, 9) is 1.

> Consider the chord joining (3, 9) to the point $(3+h, (3+h)^2)$
>
> The gradient is $\dfrac{(3+h)^2-9}{h} = \dfrac{6h+h^2}{h}$
>
> When $h = 0$ the gradient is $\dfrac{0}{0}$ so the gradient of the curve is 1

**(a)** Identify a fault in the argument. **[1]**

**(b)** Write a valid first principles argument leading to the correct value for the gradient at (3, 9). **[3]**

**(c)** Find the equation of the normal to the curve at the point (3, 9). **[2]**

**END OF QUESTION PAPER**